UNSUPERVISED STATISTICAL SEGMENTATION OF MULTISPECTRAL SAR IMAGES USING GENERALIZED MIXTURE ESTIMATION

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Abstract

This work deals with the estimation of generalized mixtures with applications to unsupervised statistical multisensor image segmentation. A mixture is said to be "generalized" when the exact nature of the noise components is not known; one assumes, however, that each belongs to a finite known set of families of distributions. We propose some methods of estimation of such mixtures based on Expectation-Maximization (EM), and Iterative Conditional Estimation (ICE, [6]) algorithms. The set of families of distributions is assumed to lie in Pearson's system.

1. INTRODUCTION

It is well established that statistical methods of segmentation can show exceptional efficiency. One can distinguish global methods, which use Markovian models [1], [3], [4], [6], and local ones [5], [8]. When unsupervised segmentation is concerned, one has to estimate the required model parameters in a previous step. The corresponding statistical problem is that of mixture estimation, and techniques like EM or SEM [5] can generally be used. In "classical" mixtures the nature of the noise distributions is known: for instance, they are all Gaussian, or Gamma, or Beta, etc. In real situations it can happen that this nature differs with the class. Furthermore, in the multisensor case, it can differ with the sensor for a given class. Moreover, for a given class and a given sensor this nature can vary in time. Thus it would be very useful to be able to automatically find the right nature of the distribution for each class and each sensor. Pioneer results of such works are presented in [7].

Our work addresses the problem of a generalized multispectral mixture estimation with application to unsupervised segmentation of SAR images. A mixture is said to be "generalized" when the exact nature of the noise components is not known; we assume, however, that each belongs to a finite known set of families of distributions. For instance, in the case of three classes and two sensors, if each component can be exponential or Gaussian, there are thirty-six possibilities of "classical" mixture. Thus the observed process is a distribution mixture and the problem of estimating such a "generalized" mixture contains a supplementary difficulty: one has to label, for each class and each sensor, the exact nature of the corresponding distribution.

When considering both global and local methods, classical mixture estimation algorithms such as EM, ICE, and SEM, can be adapted to such situations. Among different possibilities, we describe one of the generalized mixture estimation methods, valid in the context of Hidden Multisensor Markov Fields, using the Pearson system and ICE.

Different algorithms are then applied to the problem of unsupervised Bayesian multispectral SAR image segmentation. We propose an adaptive version of SEM in the case of "blind", i.e., "pixel by pixel", segmentation and compare its efficiency to the global ICE based method.

2. PEARSON'S SYSTEM

In this section we specify the set of families $\Phi = \{F_1, \dots, F_8\}$ we will use in the unsupervised radar image segmentation. Our description of Pearson's system is brief as further details can be found in [2].

A probability density function f on R belongs to Pearson's system if it satisfies :

$$\frac{1}{f(y)}\frac{df(y)}{dy} = -\frac{y+a}{c_0 + c_1 y + c_2 y^2} \quad (1)$$

The variation of the parameters a, c_0, c_1, c_2 provides distributions of different shape and, for each shape, defines the parameters fixing a given distribution. Let Y be a real random variable whose distribution belongs to Pearson's system. For $q = 1,2,3,4$ let us consider the moments of Y defined by :

$$\mu_1 = E[Y] \qquad \mu_q = E[(Y - E(Y))^q] \quad \text{for } q \geq 2 \quad (2)$$

and two parameters γ_1, γ_2 defined by :

$$\gamma_1 = \frac{(\mu_3)^2}{(\mu_2)^3} \qquad \gamma_2 = \frac{\mu_4}{(\mu_2)^2} \tag{3}$$

$\sqrt{\gamma_1}$ is called "skewness" and γ_2 "kurtosis".

On the one hand, the coefficients a, c_0, c_1, c_2 are then linked with $\mu_1, \mu_2, \gamma_1, \gamma_2$ by the following formula :

$$a = \frac{(\gamma_1 - \gamma_2 + 1)\mu_1 + (\gamma_2 + 3)\sqrt{\gamma_1 \mu_2}}{10\gamma_2 - 12\gamma_1 - 18}$$

$$c_0 = \frac{\mu_2(4\gamma_2 - 3\gamma_1)}{10\gamma_2 - 12\gamma_1 - 18} \quad c_1 = \frac{\sqrt{\gamma_1 \mu_2}(\gamma_2 + 3)}{10\gamma_2 - 12\gamma_1 - 18} \tag{4}$$

$$c_2 = \frac{(2\gamma_2 - 3\gamma_1 - 6)}{10\gamma_2 - 12\gamma_1 - 18}$$

On the other hand, putting

$$\lambda = \frac{\gamma_1(\gamma_2 + 3)^2}{4(4\gamma_2 - 3\gamma_1)(2\gamma_2 - 3\gamma_1 - 6)} \tag{5}$$

the eight families of the set $\Phi = \{F_1, \ldots, F_8\}$ are defined by :

$$\begin{aligned}
&[P_Y \in F_1] \Leftrightarrow [\lambda \langle 0] \quad [P_Y \in F_2] \Leftrightarrow [\gamma_1 = 0 \, and \, \gamma_2 \langle 3] \\
&[P_Y \in F_3] \Leftrightarrow [2\gamma_2 - 3\gamma_1 - 6 = 0] \\
&[P_Y \in F_4] \Leftrightarrow [0 \langle \lambda \langle 1] \quad [P_Y \in F_5] \Leftrightarrow [\lambda = 1] \\
&[P_Y \in F_6] \Leftrightarrow [\lambda \rangle 1] \\
&[P_Y \in F_7] \Leftrightarrow [\gamma_1 = 0 \, and \, \gamma_2 \rangle 3] \\
&[P_Y \in F_8] \Leftrightarrow [\gamma_1 = 0 \, and \, \gamma_2 = 3]
\end{aligned} \tag{6}$$

The exact form of different densities can be seen in [2]. In particular we have, F_1: beta distributions of the first kind; F_3: gamma distributions; F_5: Inverse gamma distributions; F_6: beta distributions of the second kind; and F_8: Gaussian distributions.
Note that the moments $\mu_1, \ldots, \mu_4$ can be easily estimated by empirical moments, from which we deduce the estimated values of γ_1, γ_2 by (3) and, finally, we estimate the family using (6).

3. GENERALIZED ICE

Let us briefly describe how generalized ICE runs in the context of Multisensor Hidden Markov Fields. For a set of pixels S, we consider two sets of random variables $X = (X_s)_{s \in S}$, $Y = (Y_s)_{s \in S}$ called "random fields". Each X_s takes its values in a finite set of classes $\Omega = \{\omega_1, \ldots, \omega_m\}$ and each Y_s takes its values in R^m. The field X is Markovian and we will denote by α all parameters defining its distribution P_X. The random variables $(Y_s)_{s \in S}$ will be assumed independent conditionally to X, and furthermore, the distribution of each Y_s conditional to X will be assumed equal to its distribution conditional on X_s. Under these hypotheses all distributions of Y conditional to X are defined by the k distributions of Y_s conditional to $X_s = \omega_1, \ldots, \omega_k$ respectively, whose densities will be denoted by $f_1, \ldots, f_k$. The sensors will be assumed independent, which implies

$$f_i(y_s) = f_i(y_s^1, \ldots, y_s^m) = f_{i1}(y_s^1) \times \ldots \times f_{im}(y_s^m) \tag{7}$$

Note that realizations of X according to its posterior distribution are possible (Gibbs sampler).
Thus each f_{ij} lies in one of the eight families of Pearson's system and the problem is to find each of them. We assume that we dispose of an estimator $\hat{\alpha} = \hat{\alpha}(X)$ of the parameters α.
The ICE-PEAR is an iterative procedure which runs as follows:
(i) Initialize the procedure in some way. For instance, take all f_{ij} Gaussian with parameters estimated by some classical algorithm.
(ii) Calculate $(\alpha^{q+1}, f_{ij}^{q+1})$ from $Y = y$ and (α^q, f_{ij}^q) in the following way:
(a) Simulate x^q, a realization of X according to its α^q and $f_1^q, \ldots, f_k^q$ based distribution conditional to $Y = y$.
(b) Calculate $\alpha^{q+1} = \hat{\alpha}(x^q)$.
(c) For $i = 1, \ldots, k$, consider $S_i = \{s \in S / x_s^q = \omega_i\}$. For each sensor j calculate the four first moments from $y_i^j = (y_s^j)_{s \in S_i}$ and decide, using (3), (5), and (6), in which family among $F_1, \ldots, F_8$ the distribution f_{ij} lies. Use (4) in order to calculate the parameters.
(d) Consider the densities f_{ij} found in (c) as (f_{ij}^{q+1})
It is possible to propose an analogous "blind", i.e. "pixel by pixel" SEM based algorithm and its adaptive version, in which priors depend on pixels [7].

4. EXPERIMENTS

4.1 Synthetic image

Let us consider a synthetic image "Ring" below and its noisy versions in three sensors. ICE-PEAR is the result of the Maximum Posterior Mode (MPM [4]) segmentation based on ICE-PEAR estimates and Adaptive Generalized SEM designates the result of the classical local Bayesian segmentation based on estimates with Adaptive Generalized SEM .

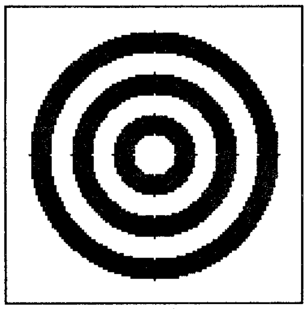
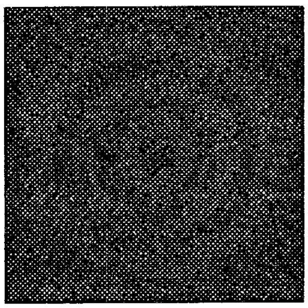
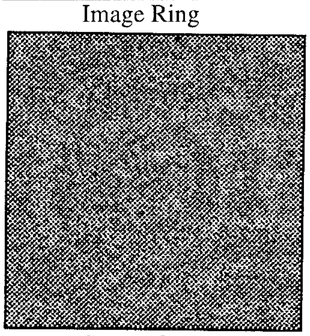


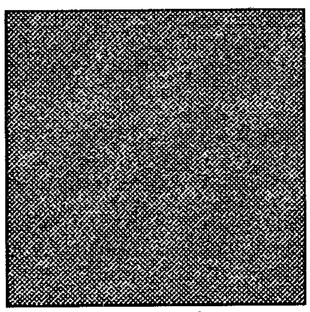
Image Ring



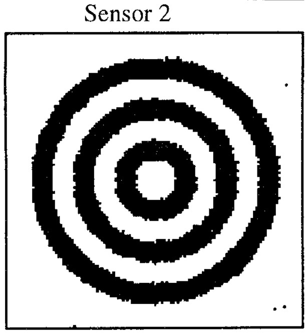
Sensor 1



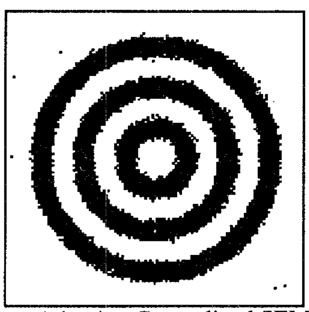
Sensor 2



Sensor 3

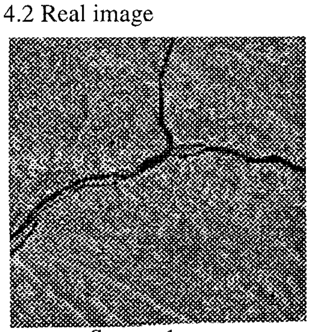


ICE-PEAR

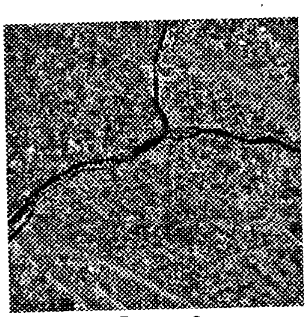


Adaptive Generalized SEM

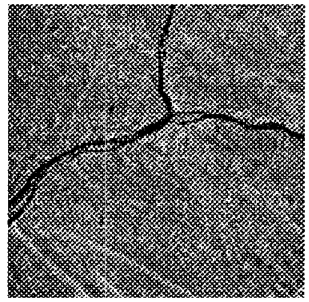
4.2 Real image



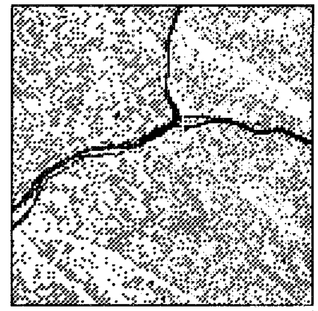
Sensor 1



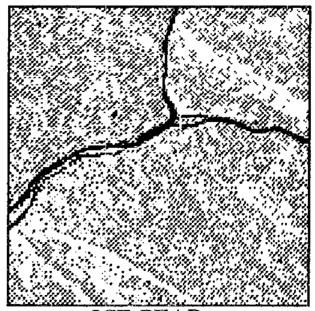
Sensor 2



Sensor 3



Generalized SEM



ICE-PEAR

Adaptive Generalized SEM

5.CONCLUSION

Different generalized mixture estimation algorithms allow one to find automatically the correct form of the noise for each class and each sensor, which allow one to generalize the classical unsupervised image segmentation methods.

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