Unsupervised Segmentation of SAR Images Using Gaussian Mixture-Hidden Evidential Markov Fields

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***Abstract*— Hidden Markov fields have been extensively applied in the field of synthetic aperture radar (SAR) image processing, mainly for segmentation and change detection. In such models, the hidden process of interest X is assumed to be a Markov field that is to be searched from an observable process Y. The possibility of such estimation lies, however, on several assumptions that turn out to be unsuitable for many natural systems. These models have then been extended in many directions, leading to triplet Markov fields among other extensions. A link has then been established between these models and the theory of evidence, opening new possibilities of uncertainty modeling and information fusion. The aim of this letter is to further generalize the hidden evidential Markov field (EMF) to consider more general forms of noise with application to unsupervised segmentation of SAR images. For parameters estimation, we use iterative conditional estimation, whereas maximization is performed through iterative conditional mode. The performance of the proposed model is assessed against the original EMF on real SAR images.**

***Index Terms*— Hidden Markov fields (HMFs), non-Gaussian noise, triplet Markov fields (TMFs), unsupervised segmentation.**

I. Introduction

SYNTHETIC aperture radar (SAR) may be the only useful data in some circumstances, covering emergency rescue management, due to their ability to function by night and under cloudy conditions. In spite of the existence of higher resolution sensors, associated images may prove unhandy in very large areas monitoring given their computational burden and the unnecessary details they catch. In this context, many statistical approaches have been proposed for classification of SAR and remote sensing imagery [1]. In particular, hidden Markov fields (HMFs) [2]–[4] have been extensively used in this area [5]–[8]. Their notoriety stems mainly from their ability to find optimal Bayesian solutions within reasonable time [9]. Let S be the set of image pixels, with $|S| = N$, and let $(Y_s)_{s\in S}$ and $(X_s)_{s\in S}$ be two random fields, where Y is observable with each Y_s taking its values in $\mathbb{R}$, whereas X is hidden with each X_s taking its values from a finite set of "classes" $\Omega = \{\omega_1, \ldots, \omega_K\}$. Realizations of such fields will be denoted by lowercase letters. The problem is then to estimate $X = x$ from $Y = y$. This estimation subsumes the distribution of (X, Y) to be known in advance. The most common way to do so is to define: 1) the distribution of X, called "prior" distribution and 2) the distribution of Y conditional on X, called "noise" distribution.

HMFs have been generalized in many ways. One particular extension that will be useful for the sake of this letter is the one leading to "triplet Markov fields" (TMFs), in which a third finite discrete valued random field $(U_s)_{s\in S}$ is introduced and the triplet (U, X, Y) is assumed Markov [10], [11]. While TMFs generalize HMFs, Bayesian processing can still be performed using the same techniques, and hence, with comparable computational complexity. The "auxiliary" field U has been assigned two main meanings.

1) To model the nonstationary aspect of the hidden field X, either in the Bayesian context by considering the realizations of U_s as the different stationarities of X as in [11]; or by using U to model the uncertainty attached to the prior knowledge of X in accordance with Dempster–Shafer (DS) theory of evidence as in [12].
2) To approximate the noise distributions, whose form is not necessarily known, by a Gaussian mixture [10].

Considering both interpretations simultaneously, we propose a Markov field model called Gaussian mixture-hidden evidential Markov field (GM-HEMF) which integrates, on the one hand, an auxiliary field U defined in the evidential domain related to the prior field X to model its unreliability, and on the other hand, a second auxiliary field U', modeling the unknown form of noise distributions $p(y_s|x_s)$. The aim of this letter is to show that such a model improves the segmentation results obtained by the "simple" HEMF proposed in [12] on real SAR images.

Let us point out that the use of theory of evidence [13], [14] within Markov models in general, and fields in particular, is recent. It stems from the fact that DS fusion can be perceived as an extension of the probabilistic computation of the "*a posteriori*" distribution required for statistical inference of the hidden field X, and hence, this computation can still be achieved in more general contexts dealing with more general measures than probabilistic masses. The interest of combining DS theory and HMFs has been established in [5], [12], and [15]–[18]. Tupin *et al.* [16] use DS fusion of several structure detectors for automatic interpretation of SAR images. Note that the theory of evidence has also been used in the Markov chains context for image modeling problems [19]–[22].

The remainder of this letter is organized as follows. Section II recalls the theory of evidence, and its use within HMFs. The proposed GM-HEMF is described in

Manuscript received June 20, 2016; revised September 1, 2016; accepted October 2, 2016. Date of publication October 21, 2016; date of current version December 7, 2016.

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Digital Object Identifier 10.1109/LGRS.2016.2615647

Section III. Section IV provides the experimental results obtained on real SAR images. The concluding remarks and future directions end this letter.

II. Markov Fields and Theory of Evidence

In this section, we briefly recall the theory of evidence and discuss its applicability to generalize Markov field models.

A. Theory of Evidence

Let $\Omega = \{\omega_1, \dots, \omega_K\}$ be a frame of discernment, and let $P(\Omega) = \{A_1, \dots, A_q\}$ be its associated powerset, with $q = 2^K$. A "basic belief assignment" (*bba*) is a function M from $P(\Omega)$ to $[0, 1]$ verifying $M(\emptyset) = 0$ and $\sum_{A \in P(\Omega)} M(A) = 1$. A *bba* M defines then a "plausibility" function Pl from $P(\Omega)$ to $[0, 1]$ by $Pl(A) = \sum_{A \cap B \neq \emptyset} M(B)$, and a "credibility" function Cr from $P(\Omega)$ to $[0, 1]$ by $Cr(A) = \sum_{B \subset A} M(B)$. For a given *bba* M, the corresponding plausibility function Pl and credibility function Cr are linked by $Pl(A) + Cr(A^c) = 1$. A probability function p can then be seen as a particular case in which $Pl = Cr = p$. Furthermore, when two *bba*s M_1 and M_2 represent two pieces of evidence, we can combine, or fuse, them using the so-called "DS fusion" (DS fusion), which gives $M = M_1 \oplus M_2$ defined as $M(A) = (M_1 \oplus M_2)(A) \propto \sum_{B_1 \cap B_2 = A \neq \emptyset} M_1(B_1) M_2(B_2)$ for any $A \neq \emptyset$. Finally, we will say that a *bba* is "probabilistic" or "Bayesian" when it is null outside singletons, and we will say that it is "evidential" otherwise.

B. Hidden Markov Fields

In the HMFs context, the field X is assumed Markovian with respect to the system of cliques C, associated with a neighborhood system $\mathfrak{N} = (\mathfrak{N})_{s \in S}$, X is then called a Markov random field (MRF) defined as

$$p(x) \propto \exp\left[-\sum_{c \in C} \phi_c(x_c)\right] \tag{1}$$

where ϕ_c is a potential function associated with clique c.

To define the distribution of Y conditional on X, two assumptions are usually set: 1) the random variables $(Y_s)_{s \in S}$ are independent conditional on X and 2) the distribution of each Y_s conditional on X is equal to its distribution conditional on X_s. The noise distribution is then fully defined through K distributions $(f_i)_{1 \leq i \leq K}$ on $\mathbb{R}$, where f_i denotes the density of the distribution of Y_s conditional on $X_s = \omega_i$. Since $p(x, y) = p(x) p(y|x)$, we obtain

$$p(x, y) \propto \exp -\left[\sum_{c \in C} \phi_c(x_c) - \sum_{s \in S} \log f_{x_s}(y_s)\right]. \tag{2}$$

Hence, according to (2), the couple (X, Y) is a Markov field and is also the distribution of X conditional on $Y = y$. This allows to sample a realization of X according to its posterior distribution $p(x|y)$ and hence, to apply Bayesian techniques like maximum posterior marginal and maximum *a posteriori* (MAP). HMFs have then been extended in many directions. One interesting extension is that leading to TMFs [10], [11]. Extending HMFs to TMFs consists in introducing a third process $U = (U_s)_{s \in S}$, where each U_s takes its values in a finite set $\Lambda = \{\lambda_1, \dots, \lambda_J\}$, and considering that $T = (U, X, Y)$ is a Markov field

$$p(t) = \gamma \exp\left[-\sum_{c \in C} \phi_c(t_c)\right] \tag{3}$$

where γ is a normalizing constant.

The conventional processing methods to estimate X from $Y = y$ remain workable. As specified in [10], the auxiliary random field U can have different meanings and its estimation, which is also possible, may be of interest. As stated in Section I, U can model the fact that the field X may be nonstationary, which may be of interest, particularly in textured image segmentation [11].

C. Hidden Evidential Markov Fields

Let us consider the random fields $X = (X_s)_{s \in S}$, $Y = (Y_s)_{s \in S}$ and let

$$p_1(x) \propto \exp\left[-\sum_{c \in C} \phi_c(x_c)\right]$$

and

$$p^y(x) = \frac{\prod_{s \in S} f_{x_s}(y_s)}{\sum_x \prod_{s \in S} f_{x_s}(y_s)}.$$

Then the posterior distribution $p(x|y)$ associated with the HMF given by (2) can be seen as the DS fusion of p_1 and p^y: $p(x|y) = (p_1 \oplus p^y)(x)$. That is of importance as it opens the way to different possibilities of extensions [12]. More precisely, if either p_1 or p^y is extended in $p_1 \oplus p^y$ to an evidential *bba*, the fusion result remains a probability distribution, which can then be seen as an extension of the classic posterior probability $p(x|y)$. Furthermore, if the "evidential" extension of p_1 or p^y is of a similar Markovian form, in spite of the fact that the fusion result is no longer necessarily a Markov field, the computation of posterior margins $p(x_s|y)$ remains feasible.

For instance, if p_1 is replaced by a Markov *bba* M, called evidential Markov field (EMF [12]) defined on $P(\Omega)^N$ by

$$M(u) \propto \exp\left[-\sum_{c \in C} \phi_c(u_c)\right] \tag{4}$$

then, the DS fusion $M \oplus p^y$ is the posterior distribution $p(x|y)$ associated with $p(x, y)$, which is itself a marginal distribution of a TMF $T = (U, X, Y)$ and hence X can still be estimated from $Y = y$ [12].

The usefulness of such extensions has already been established in hidden nonstationary Markov fields with application to image segmentation [12]. Indeed, when the prior distribution p_1 is nonstationary, replacing it with a stationary EMF M of (4) form can provide better performance, in unsupervised segmentation, than replacing it with any other stationary classic HMF. This is particularly of interest in the unsupervised context, where all the parameters have to be estimated from the observation $Y = y$. Using some estimation method leads, when keeping the classic model given by (2), to a stationary $\hat{p}_1$. When using a stationary extension of (4) form, the model parameters can still be estimated from $Y = y$ and it turns out that $\hat{M}$-based segmentation can provide significantly better results. Such models are called HEMFs.

III. Gaussian Mixture-Hidden Evidential Markov Fields

In this section, we describe the proposed GM-HEMF model that generalizes the HEMF proposed in [12]. For this purpose, let us consider the fields $X = (X_s)_{s\in S}$ and $Y = (Y_s)_{s\in S}$ of Section II-C. As stated above, if we define p_1 from the MRF $p(x) \propto \exp[-\sum_{c\in C} \phi_c(x_c)]$ and p^y from the simple $p(y|x) = \prod_{s\in S} f_{x_s}(y_s)$, then the DS fusion $(p_1 \oplus p^y)(x)$ is itself the posterior distribution $p(x|y)$. Extending p_1 to the EMF M defined on $P(\Omega)^N$ by $M(u) \propto \exp[-\sum_{c\in C} \phi_c(u_c)]$ leads to HEMFs. If further, the distributions $f_{x_s}(y_s)$ are Gaussian, the model is then called "Gaussian noise-HEMF." In this section, we propose to generalize this latter by considering some more general noise forms. To this end, let us extend $f_{x_s}(y_s)$ to a Gaussian mixture. Hence we have

$$p(y_s|x_s) = \sum_{j=1}^{J} \alpha_{x_s,j} f_{x_s,j}(y_s) \tag{5}$$

where $\alpha_{x_s,j}$ is the weight of the jth component associated with class x_s.

Let $U' = (U'_s)_{s\in S}$ be an auxiliary field with each U'_s taking its values in $\Lambda = \{\lambda_1, \ldots, \lambda_J\}$. Then (5) can be written as

$$p(y_s|x_s) = \sum_{u'_s\in\Lambda} \alpha_{x_s,u'_s} f_{x_s,u'_s}(y_s) = \sum_{u'_s\in\Lambda} p\left(y_s, u'_s|x_s\right). \tag{6}$$

Then, p^y can be extended to the probabilistic *bba* Q^y given by

$$Q^y(x, u') \propto \prod_{s\in S} \alpha_{x_s,u'_s} f_{x_s,u'_s}(y_s). \tag{7}$$

Proposition 1: The DS fusion $(M \oplus Q^y)(x)$ defines a Markov field given by

$$p(u, x, u', y) \propto \exp\left[-\sum_{c\in C} \varphi_c\left(u_c, x_c, u'_c, y_c\right)\right]$$

where

$$\varphi_c(u_c, x_c, u'_c, y_c) = \phi_c(u_c) - \sum_{s\in c} \log(\alpha_{x_s,u'_s}) - \sum_{s\in c} \log(f_{x_s,u'_s}(y_s)).$$

Proof:

$$\begin{aligned}
&(M \oplus Q^y)(x) \\
&\quad\propto \sum_{u\ni x,u'} \exp\left[-\sum_{c\in C}\phi_c(u_c)\right]\prod_{s\in S}\alpha_{x_s,u'_s} f_{x_s,u'_s}(y_s) \\
&\quad\propto \sum_{u\ni x,u'} \exp\left[-\sum_{c\in C}\phi_c(u_c) + \sum_{s\in S} log[\alpha_{x_s,u'_s} f_{x_s,u'_s}(y_s)]\right] \\
&\quad\propto \sum_{u\ni x,u'} \exp\left[-\sum_{c\in C}\varphi_c\left(u_c, x_c, u'_s, y_c\right)\right].
\end{aligned}$$

□

Definition 1: Let us consider the following:

1) $\Omega = \{\omega_1, \ldots, \omega_K\}$ a set of classes, and $P(\Omega)$ its associated powerset;
2) $\Lambda = \{\lambda_1, \ldots, \lambda_J\}$ a finite set;
3) S a set of pixels with $|S| = N$, and $V = (V_s)_{s\in S} = (U, X, U', Y) = (U_s, X_s, U'_s, Y_s)_{s\in S}$ a random field, each (U_s, X_s, U'_s, Y_s) taking its values in $\Delta \times \Lambda \times \mathbb{R}$ where $\Delta = \{(A, \omega) \in P(\Omega) \times \Omega | \omega \in A\}$.

Then V is called GM-HEMF if its distribution is given on $[\Delta \times \Lambda \times \mathbb{R}]^N$ by

$$p(v) \propto \exp\left[-\sum_{c\in C}\phi_c(u_c) - \sum_{s\in S}\eta_s\left(x_s, u'_s\right) + \sum_{s\in S}\log\left(p\left(y_s|x_s, u'_s\right)\right)\right]$$

where C is the set of cliques related to some neighborhood system and η_s is a potential function associated with site s.

Let us show how the proposed GM-HEMF generalizes some classic Markov fields. Reducing Λ to a singleton leads to the former HEMF introduced in [12]. If further, $U = X$ one finds again the classic HMF.

In practice, there is no theoretical difficulty to estimate the model's parameters from the sole observation y once the form of potentials are chosen, which makes the model unsupervised. For instance, one can apply "iterative conditional estimation" as in [10], [11], and [23]. Let $Z = (X, U, U')$. Given a set of parameters θ^q, Gibbs sampler is used to produce a classification z^{q+1}, from which a new set of parameters θ^{q+1} is estimated. The same process is then repeated until a certain criterion is satisfied, such as $||z^q - z^{q-1}|| < \varepsilon$ for a certain fixed value of ε (here $\varepsilon = 50$). Then, the Bayesian MAP method is approximated by the classic "iterative conditional mode" algorithm [2]. The initial set of parameters θ^0 required to start this iterative procedure can be obtained as follows: the image y is first segmented through an automatic histogram thresholding. Then, likelihood parameters are estimated via classic estimators. Finally, prior parameters are estimated through least square fitting. Please refer to the flowchart in Fig. 1.

IV. Experiments

In this section, we assess the performance of the proposed GM-HEMF against both HMF and HEMF models. To this end, we consider two real SAR images. Image 1 is a moderately noisy 256×256 image taken by the Jet Propulsion Laboratory on L band [24] [see Fig. 2(a)]. Image 2 is a JERS1 SAR image of Rondonia, Brazil, a heavily noisy 330×512 image [see Fig. 2(b)]. Let S be the set of image pixels. Then, we have an observed image $Y = (Y_s)_{s\in S}$ with $|S| = 65\,536$ for Image 1 and $|S| = 168\,960$ for Image 2. The ground-truth images associated with Images 1 and 2 are also provided in Fig. 2(a).2 and (b).2, respectively. Let $X = (X_s)_{s\in S}$ where each $X_s \in \Omega = \{\omega_1, \ldots, \omega_K\}$ with $K = 2$ for Image 1 and $K = 3$ for Image 2; and let $U = (U_s)_{s\in S}$ where each $U_s \in P(\Omega)$. Finally, let $U' = (U'_s)_{s\in S}$ with each U'_s taking its values from the finite set $\Lambda = \{\lambda_1, \ldots, \lambda_J\}$ with J being the number of Gaussian components.

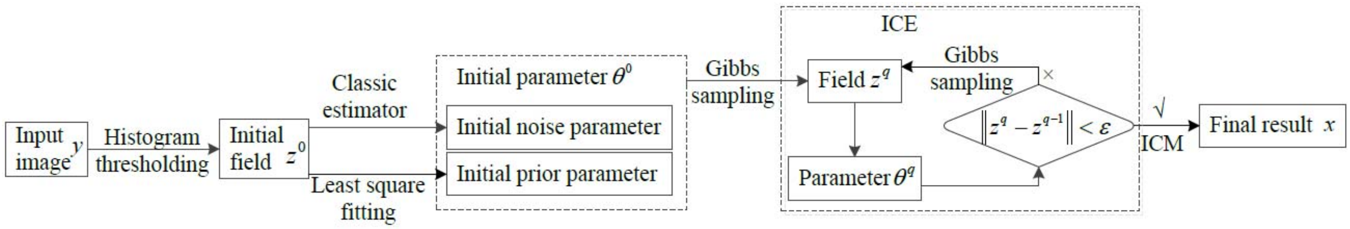


Fig. 1. Different steps in the proposed approach.

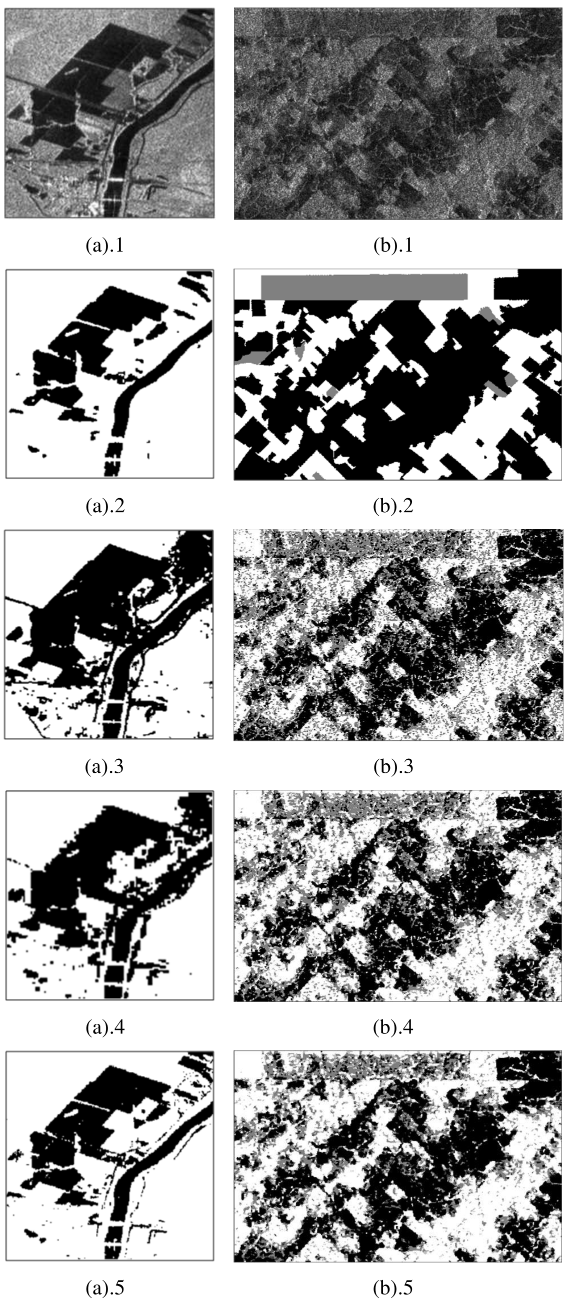


Fig. 2. Unsupervised segmentation of noisy SAR images. 1—real data; 2—ground truth; 3—HMF-based result; 4—HEMF-based result; and 5—GM-HEMF-based result of (a) Image 1 and (b) Image 2.

Then, unsupervised segmentation is conducted according to the following three models: the HMF (X, Y), the HEMF (U, X, Y), and the GN-HMF (U, X, U', Y), with $J = 5$.

TABLE I

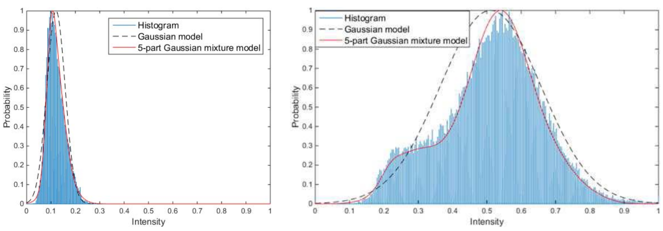
Performance Evaluation of Different Models on SAR Image Segmentation [τ: Overall Accuracy (%), κ: Kappa Metric]

Image	HMF		HEMF		GM-HEMF	
	τ	κ	τ	κ	τ	κ
Image 1	81.45	0.598	85.11	0.664	96.75	0.915
Image 2	58.36	0.369	62.13	0.413	65.76	0.453

The results obtained are illustrated in Fig. 2. In Table I, the segmentation quality of different models is always assessed quantitatively in terms of the overall accuracy (τ) and the comprehensive Kappa metric (κ).

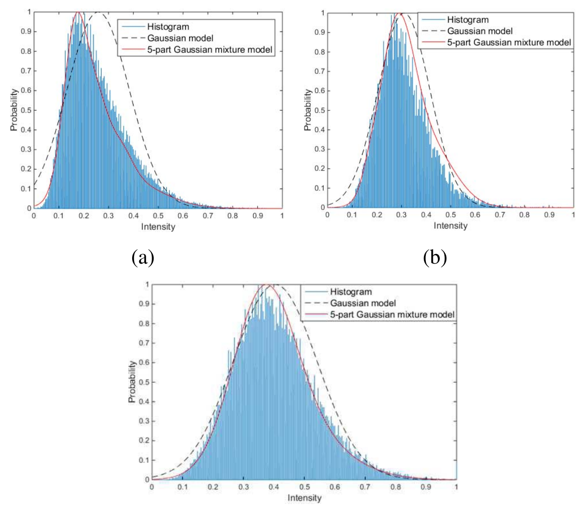
A visual assessment of the segmentation results, provided in Fig. 2, establishes the superiority of the proposed GM-HEMF over both the former HMF and HEMF models. For Image 1, the obtained segmentation is very close to the ground truth. For Image 2, which is much noisier, the proposed model furnishes the best segmentation in terms of homogeneity and edge preserving. The quantitative evaluation of the models' performance, provided in Table I, confirms the interest of the proposed GM-HEMF with respect to HMF and HEMF. Indeed, in terms of both the overall accuracy and Kappa metrics, the proposed model yields the best result. For Image 1, the difference is even striking ($\tau_{\text{GM-HEMF}} = 96.75\%$ against $\tau_{\text{HEMF}} = 85.11\%$ and $\tau_{\text{HMF}} = 81.45\%$). Derrode and Pieczynski [25] also considered Image 2 using copulas with pairwise Markov chains. The overall accuracy obtained was $\tau = 62.40\%$, whereas our model yields $\tau_{\text{GM-HEMF}} = 65.76\%$. Let us underline that when data are too noisy (as for Image 2), there may be no way to obtain satisfying performance. To overcome such drawbacks, hyperspectral/multisensor data are generally used to weaken the impact of noise.

To better understand these results, Fig. 3 (respectively, Fig. 4) depicts for each class of Image 1 (resp. Image 2): the histogram of the actual image intensity, the estimated Gaussian distribution, and the estimation of the Gaussian mixture distribution with $J = 5$. As can be checked visually, the Gaussian mixture distribution is better suited to fit the actual noise density, especially for ω_2 of Image 1 for which, both HMF and HEMF perform poorly when characterizing pixels that have approximate distances to both the classes. Increasing the number of Gaussian components makes it possible to fit these pixels appropriately. In real circumstances, it is difficult to know the genuine form of noise densities. Gaussian distributions are thus often adopted to model the data histogram. However, when the actual noise distribution



(a) (b)

Fig. 3. Actual intensity histogram (blue line), the estimated Gaussian distribution (black dashed line), and mixture Gaussian distribution with $J = 5$ (red solid line) in Image 1. (a) Class ω_1. (b) Class ω_2.



(a) (b) (c)

Fig. 4. Actual intensity histogram (blue line), the estimated Gaussian distribution (black dashed line) and mixture Gaussian distribution with $J = 5$ (red solid line) in Image 2. (a) Class ω_1. (b) Class ω_2. (c) Class ω_3.

is not Gaussian, which occurs with many natural systems, such an assumption turns out to be inappropriate, and for this reason, the GM-HEMF is better suited than the former HEMF. In theory, mixture Gaussian distribution can fit any type of noise as long as there are enough Gaussian models within it.

V. Conclusion

In this letter, we have proposed a new approach for unsupervised segmentation of SAR images based on GM-HEMF, which generalizes, in particular, the HEMF. The interest of the proposed model relies in its ability to fit some general forms of noise that the former HEMF fails to support. This property has been checked through experiments carried out on real SAR images. A promising future direction would be to consider more complicated noise forms by extending the noise energies to cliques rather than defining them on single pixels.

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