Unsupervised Segmentation of Markov Random Fields Corrupted by Nonstationary Noise

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***Abstract*—Hidden Markov fields have been widely used in image processing thanks to their ability to characterize spatial information. In such models, the process of interest X is hidden and is to be estimated from an observable process Y. One common way to achieve the associated inference tasks is to define, on one hand, the prior distribution $p(x)$; and on the other hand, the noise distribution $p(y|x)$. While it is commonly established that the prior distribution is given by a Markov random field, the noise distribution is usually given through a set of Gaussian densities; one per each label. Hence, observed pixels belonging to the same class are assumed to be generated by the same Gaussian density. Such assumption turns out, however, to be too restrictive in some situations. For instance, due to light conditions, pixels belonging to a same label may present quite different visual aspects. In this letter, we overcome this drawback by considering an auxiliary field U in accordance with the triplet Markov field formalism. Experimental results on simulated and real images demonstrate the interest of the proposed model with respect to the common hidden Markov fields.**

***Index Terms*—Hidden Markov fields (HMFs), nonstationary noise, triplet Markov fields (TMFs), unsupervised segmentation.**

I. Introduction

HIDDEN Markov fields (HMFs) have been successfully employed to solve problems encountered in image processing [1]–[6]. Their notoriety stems mainly from their ability to find optimal Bayesian solutions within reasonable time. Let S be the set of image pixels, with $|S| = N$, and let $(X_s)_{s\in S}$ and $(Y_s)_{s\in S}$ be two random fields where X is hidden with each X_s taking its values from a finite set $\Omega = \{\omega_1, \dots, \omega_K\}$, called set of "classes" or "labels"; whereas Y is observable with each Y_s taking its values in $\mathbb{R}$ (or $\mathbb{R}^d$). Realizations of such fields will be denoted using lowercase letters and we will write $p(x)$ instead of $p(X = x)$ for simplicity sake. Then, the problem is to estimate $X = x$ from $Y = y$. For this, the joint distribution of the couple (X, Y) is assumed known in advance. To do so, one usually defines, on one hand, the prior distribution $p(x)$, and on the other hand, the likelihood distribution $p(y|x)$, also called "noise" distribution. In HMFs context, the hidden field X is assumed Markovian with respect to some neighborhood system $\mathcal{N} = (\mathcal{N})_{s\in S}$. X is then called a Markov random field (MRF) defined by $p\left(x_s|x_{S-\{s\}}\right) = p\left(x_s|x_{\mathcal{N}_s-\{s\}}\right)$, that can be rewritten considering the Hammersley–Clifford equivalence as follows:

$$p(x) = \frac{1}{Z}\exp[-W(x)] \propto \exp\left[-\sum_{c\in C}\phi_c(x_c)\right] \qquad (1)$$

where ϕ_c are potential functions defined with respect to a system of cliques C associated to $\mathcal{N}$. To define the noise distribution, two assumptions are usually set: (i) for a given realization $X = x$, the variables $(Y_s)_{s\in S}$ are independent; and, (ii) the distributions $p(y_s|x)$ and $p(y_s|x_s)$ are equal. The noise distribution is then fully defined through K distributions $(f_i)_{1\le i\le K}$ on $\mathbb{R}$, usually assumed Gaussian: $f_i(y_s) = p(y_s|x_s = \omega_i)$. Since $p(x, y) = p(x)p(y|x)$, we obtain

$$p(x, y) = \frac{1}{Z}\exp[-W(x, y)] \propto \exp\left[-\left(\sum_{c\in C}\phi_c(x_c) - \sum_{s\in S}\log f_{x_s}(y_s)\right)\right]. \qquad (2)$$

As one can see from (2), the couple (X, Y) is a Markov field and also is the distribution of X conditional on $Y = y$. This makes it possible to sample a realization of X according to $p(x|y)$ and hence, to apply different Bayesian techniques. HMFs have been extended to "pairwise Markov fields" (PMFs), in which one directly assumes Markovianity of the pair (X, Y) [7], and "triplet Markov fields" (TMFs), in which a third finite discrete valued random field $(U_s)_{s\in S}$ is introduced and the triplet (U, X, Y) is assumed Markovian [8]. Other extensions of HMFs designed for image modeling problems include the infinite HMF model [9], formulated on the basis of a joint Dirichlet process mixture; the gated MRFs successfully applied for natural images segmentation [10], extended-likelihood-based HMF [11] considering observations correlation, and spatially adaptive state-space HMF [12] using an adaptive label set with application to cell tracing problem.

In this letter, we propose a new TMF considering the possible switches of noise distributions $p(y_s|x_s)$. More explicitly, each class ω_i is assigned a set of M noise densities $(f_i^m)_{1\le m\le M}$ rather than only one f_i. The selection of the appropriate density depends on the realization of an auxiliary Markov field U. Such model may be of particular interest in image modeling where, for instance, the image presents uneven light conditions. The same extension has been already successfully considered in the frame of hidden Markov chains with application to image segmentation [13].

The remainder of this letter is organized as follows. Section II describes the basics of the proposed TMF model. Results of experiments achieved on sampled and real images are provided

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in Section III. Finally, concluding remarks and future directions are given in Section IV.

II. Jumping-Noise HMFs

In this section, we describe a new TMF model designed for unsupervised restoration of images corrupted by nonstationary noise. To do so, we first define the problem tackled in this letter. Then, we formally define the proposed model. Finally, we demonstrate how to achieve the different associated estimation and inference tasks.

Let $Z = (X, Y)$ be an HMF given by (2). For each x_s, the distribution $p(y_s|x_s)$, denoted $f_{x_s}(y_s)$, is said stationary if it does not depend on the position of s in the set of image pixels S. Otherwise, such distribution is said nonstationary and the associated HMF is said "nonstationary noise-hidden Markov field" (NN-HMF). Such variability among noise densities may be due to many factors. Image pixels may, for instance, experience uneven light conditions. Also, the presence of shadow in some image parts may give to related pixels darker aspect [14], [15]. When the "varying" noise distributions are fully characterized, there is no theoretical difficulty to achieve the conventional labeling tasks. In the unsupervised context, however, estimation of parameters leads to a stationary HMF, where one cannot characterize the varying noise. To overcome this difficulty, we propose to express the noise densities in the following manner. For each label $\omega_i \in \Omega$, we have a family of M different noise densities, denoted $(f_i^m)_{1\leq m\leq M}$. If noise nonstationarity is due to some external factor like the ones mentioned above, there are reasons to consider such factor related to some Markov process and hence, the number of densities is the same for all $\omega_i \in \Omega$. Thus, modeling the spatial information related to such process may be of interest. Still, devoting a different M_i for each class ω_i is straightforward by simply considering $M = \max_i M_i$. In this letter, we consider an auxiliary Markov field $U = (U_s)_{s\in S}$ with each U_s taking its values in $\Lambda = \{\lambda_1, \ldots, \lambda_M\}$. The realization of each U_s will then determine the actual noise density of $X_s = x_s$ used to produce $Y_s = y_s$: $f_{x_s}^{u_s}(y_s) = p(y_s|x_s, u_s)$. The resulting TMF is called "jumping noise-hidden Markov field" (JN-HMF).

Definition 1: Let us consider:

1) $\Omega = \{\omega_1, \ldots, \omega_K\}$ a set of classes;
2) $\Lambda = \{\lambda_1, \ldots, \lambda_M\}$ a set of noise jumps;
3) S the set of image pixels;
4) $T = (T_s)_{s\in S} = (U_s, X_s, Y_s)_{s\in S}$ a random field with each T_s taking its values from $\Lambda \times \Omega \times \mathbb{R}$.

Then, the triplet $T = (U, X, Y)$ is said a jumping noise-hidden Markov field (JN-HMF) if its distribution is given by $p(u, x, y) = \frac{1}{Z}\exp[-W(u, x, y)]$ with

$$W(u,x,y) = W_\phi(x) + W_\varphi(u) - \sum_{s\in S}\log\left(f_{x_s}^{u_s}(y_s)\right)$$
$$= \sum_{c\in C}\phi_c(x_c) + \sum_{c\in C}\varphi_c(u_c) - \sum_{s\in S}\log\left(f_{x_s}^{u_s}(y_s)\right). \tag{3}$$

Furthermore, if f_i^m is Gaussian for all $\omega_i \in \Omega$ and $\lambda_j \in \Lambda$, T will be said "Gaussian" JN-HMF.

Remark 1: i) U, X are two independent Markov fields; ii) $p(x, y|u)$ is a nonstationary HMF; iii) If $\varphi_c = \varphi_{\{s\}}$ then (X, Y) is an HMF, with $p(y_s|x_s) = \sum_{u_s} p(u_s)p(y_s|u_s, x_s)$.

A. Parameters Estimation

Let $T = (U, X, Y)$ be a JN-HMF defined by (3). T is defined by the following parameters:

1) Hidden Process Parameters: For both U and X, we adopt the Potts model, which is among the most used ones [2]. We consider:

$$W_\phi(x) = \sum_{s\in S}\left[\sum_{i=1}^{K}\alpha_i\delta_{x_s}^i + \alpha_h\sum_{x_t\leftrightarrow x_s}\delta_{x_s}^{x_t} + \alpha_v\sum_{x_t\updownarrow x_s}\delta_{x_s}^{x_t}\right] \tag{4}$$

$$W_\varphi(u) = \sum_{s\in S}\left[\sum_{i=1}^{M}\beta_i\delta_{u_s}^i + \beta_h\sum_{u_t\leftrightarrow u_s}\delta_{u_s}^{u_t} + \beta_v\sum_{u_t\updownarrow u_s}\delta_{u_s}^{u_t}\right] \tag{5}$$

where:

1) $\delta_a^b = \begin{cases}1 & \text{if } a = b\\ 0 & \text{else}\end{cases}$;
2) $a \leftrightarrow b$: a is a horizontal neighbor of b;
3) $a \updownarrow b$: a is a vertical neighbor of b.

2) Noise Parameters: The likelihood distributions are defined through the set of $K \times M$ densities (f_i^m) with $1 \leq i \leq K$ and $1 \leq m \leq M$. In the Gaussian case, we deal within this letter, the noise densities are thus fully defined through $K \times M$ means and $K \times M$ standard deviations. Let θ be the parameters set of the Gaussian JN-HMF model. We have

$$\theta = (\alpha_1..\alpha_K, \alpha_h, \alpha_v, \beta_1..\beta_M, \beta_h, \beta_v, \mu_1^1..\mu_K^M, \sigma_1^1..\sigma_K^M). \tag{6}$$

We use the stochastic gradient [16], which is an iterative way, to approximate the maximum likelihood. Starting with a set of initial parameters $\theta^{(0)}$, we infer at each iteration $n+1$, the new set of parameters as follows: $\theta^{(n+1)} =$:

$$\theta^{(n)} + \frac{\kappa}{n+1}[\nabla_{\theta^{(n)}}W^{(n)}(u,x,y) - \nabla_{\theta^{(n)}}W^{(n)}(u^*,x^*)] \tag{7}$$

where κ is a constant called step size, taken in practice as $\frac{1}{N}$, $\nabla_{\theta^{(n)}}$ is the gradient at $\theta^{(n)}$.

The initial set of parameters θ^0 required to start this iterative procedure is obtained in the following way. The image is first segmented into K classes using K-means to generate a realization of X. Then within each class, another clustering is performed to segment sites according to their intensities, which provides an initial realization of U. Doing so, classic estimators can be used to estimate initial θ^0.

Thereafter, we illustrate how to iterate α_i and μ_i^j when using stochastic gradient within a Gaussian JN-HMF:

$$i \in 1..K : \alpha_i^{(n+1)} = \alpha_i^{(n)} + \frac{\kappa}{n+1}\left(\sum_{s\in S}\left(\alpha_i\delta_{x_s}^i - \alpha_i\delta_{x_s^*}^i\right)\right) \tag{8}$$

$$i \in 1..K, j \in 1..M : \mu_i^{j(n+1)} = \mu_i^{j(n)} +$$
$$\frac{\kappa}{n+1}\left(\sum_{s\in S}\left(-\frac{y_s - \mu_i^{j(n)}}{\sigma_i^{j(n)}}\delta_{(x_s,u_s)}^{(i,j)} + \frac{y_s - \mu_i^{j(n)}}{\sigma_i^{j(n)}}\delta_{(x_s^*,u_s^*)}^{(i,j)}\right)\right). \tag{9}$$

TABLE I
SEGMENTATION ERROR RATES (%) OF SIMULATED IMAGES

Experiment	K-means	HMF	JN-HMF	
	X	X	X	U
Moderate noise	25.92	16.59	4.05	4.70
Heavy noise	30.23	17.01	11.69	13.37

B. Segmentation (Restoration)

Let $T = (U, X, Y)$ be a JN-HMF as defined above and let $V = (U, X)$. Several estimators can be used to search $V = \hat{v}$ from $Y = y$, such as MAP, ICM, or MPM. This latter, used here, is defined by

$$[\hat{v} = \underset{\text{MPM}}{\hat{S}}(y)] \Leftrightarrow [\hat{v}_s = \underset{(\lambda,\omega)}{\operatorname{argmax}}\, p(v_s = (\lambda, \omega)/y)]\,. \quad (10)$$

The solution $\hat{v}$ of (10) is estimated by sampling the Markov field $(V|y)$ using, for instance, the Gibbs method [2]. Having L simulations $v^1, v^2, \ldots, v^L$ of $(V/Y = y)$, $p(v_s = (\lambda, \omega)/y)$ is estimated by

$$\hat{p}(v_s = (\lambda, \omega)/y) = \frac{1_{[u_s^1=\lambda, x_s^1=\omega]} + .. + 1_{[u_s^L=\lambda, x_s^L=\omega]}}{L}. \quad (11)$$

III. EXPERIMENTS

The aim of this section is to establish the interest of the proposed JN-HMF with respect to the conventional HMF.

A. Unsupervised Segmentation of Sampled Images

Let $T = (U, X, Y)$ be a Gaussian JN-HMF with $T = (T_s)_{s \in S}$ and $|S| = 128 \times 128$. U_s takes its values from $\Lambda = \{\lambda_1, \lambda_2\}$, X_s takes its values from $\Omega = \{\omega_1, \omega_2, \omega_3\}$, and Y_s from $\mathbb{R}$. U and X are sampled through Gibbs sampler using, respectively, the parameters $(\beta_1 = 0.0,\ \beta_2 = 0.0,\ \beta_h = 2.0,\ \beta_v = 2.0)$ and $(\alpha_1 = 0.0,\ \alpha_2 = 0.0,\ \alpha_3 = 0.0,\ \alpha_h = 2.0,\ \alpha_v = 2.0)$. Then Y is sampled with means equal to $(\mu_{\omega_1}^{\lambda_1} = 0.0,\ \mu_{\omega_1}^{\lambda_2} = 3.0,\ \mu_{\omega_2}^{\lambda_1} = 4.0,\ \mu_{\omega_2}^{\lambda_2} = 7.0,\ \mu_{\omega_3}^{\lambda_1} = 8.0,\ \mu_{\omega_3}^{\lambda_2} = 11.0)$ and standard deviations all equal to 1 in Experiment 1 (moderate noise) and equal to 2 in Experiment 2 (Heavy noise). Each experiment is conducted 100 times. Table I summarizes the average segmentation error rates whereas Fig. 1 illustrates an example of one execution (moderate noise case).

The results obtained from Experiment 1 confirm that the JN-HMF is more suitable than the conventional HMF to handle switching noise densities. For Experiment 2, the advantage of JN-HMFs over HMFs is less significant due to noise strength.

B. Unsupervised Segmentation of Synthetic Images

In this series of experiments, the realization of X is a 256×192 synthetic class-image where ω_1 and ω_2 correspond to black pixels and white ones, respectively [see Fig. 2(a).1]. Regarding the U field, we consider two cases. First, a realization of U is sampled as a Markov field [see Fig. 2(a).2]. Second, the realization of U is already given as a predefined synthetic image [see Fig. 2(b).2]. For both cases, two levels of noise will be considered (moderate and heavy) to check the JN-HMF performance with respect to the HMF one. Finally, the noise parameters are the same as those used in the first series of experiments. The results obtained are summarized in Table II. Fig. 2 illustrates the segmentation results in the moderate noise scenario for both sampled and predefined realization of U.

As shown in Table II, the proposed JN-HMF significantly outperforms the conventional HMF. From Fig. 2, one can check that while the HMF confounds both classes due to noise switches, the JN-HMF make it possible to detect and filter such switches. Another interesting observation is that the JN-HMF allows one

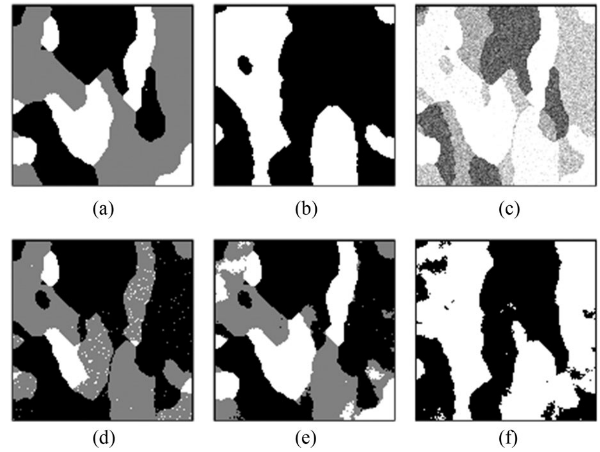

Fig. 1. Unsupervised segmentation of sampled images. (a) Original class-image $X = x$. (b) Auxiliary field realization $U = u$. (c) Noisy image $Y = y$ (d) HMF-based estimate of X. (e) JN-HMF-based estimate of X. (f) JN-HMF-based estimate of U.

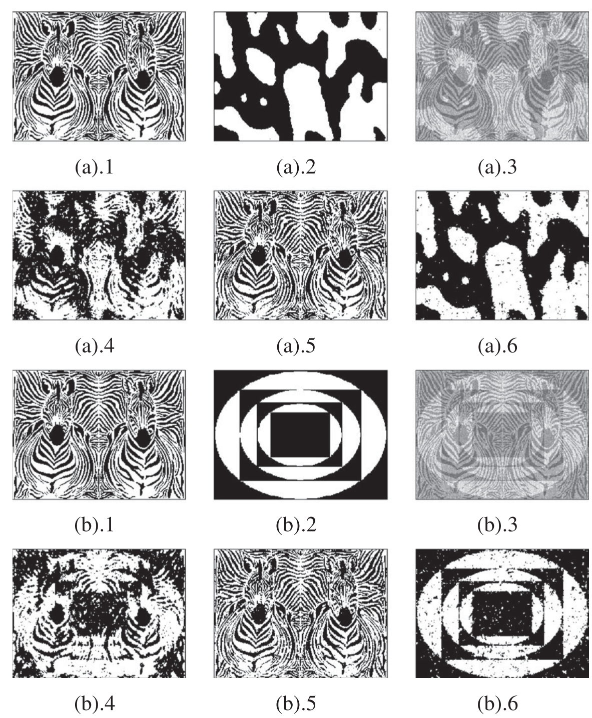

Fig. 2. Unsupervised segmentation of synthetic images. 1. Original class-image $X = x$; 2. Auxiliary field realization $U = u$; 3. Noisy image $Y = y$; 4. HMF-based estimate of X; 5. JN-HMF-based estimate of X; 6. JN-HMF-based estimate of U; of (a) Sampled auxiliary field case; (b) Predefined auxiliary field case.

TABLE II
SEGMENTATION ERROR RATES (%) OF SYNTHETIC IMAGES

	Experiment	K-means	HMF	JN-HMF	
		X	X	X	U
Sampled	Moderate noise	24.97	15.67	6.16	6.41
	Heavy noise	25.91	19.09	12.23	11.46
Predefined	Moderate noise	28.90	16.33	5.51	5.18
	Heavy noise	30.69	23.37	18.30	14.77

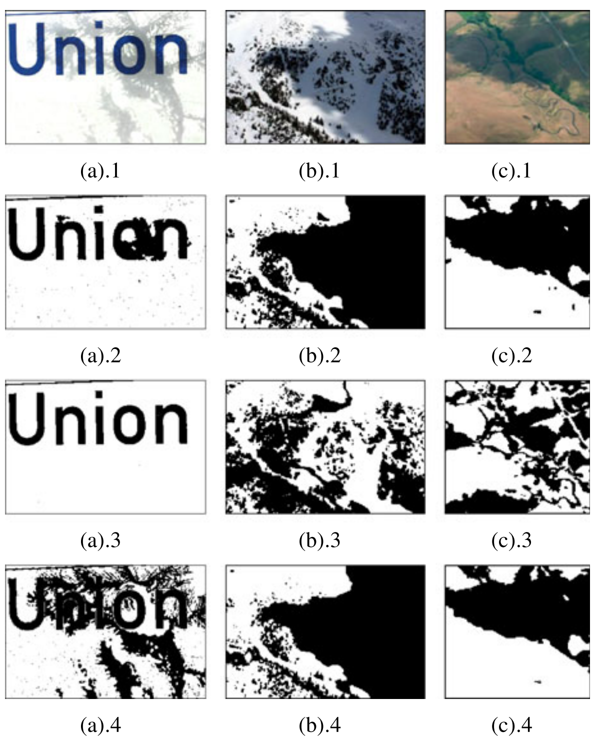

Fig. 3. Unsupervised segmentation of shadowy color images. 1. Original image; 2. HMF-based segmentation; 3. JN-HMF-based segmentation; 4. JN-HMF-based estimate of U; of (a) Image 1; (b) Image 2; (c) Image 3.

to restore the realization of the auxiliary field U, which may be of interest in many applications.

C. Unsupervised Segmentation of Real Color Images

In this series of experiments, we consider two sources of difficulty usually encountered in image segmentation: presence of shadow (Images 1–3 of Fig. 3); and uneven light conditions (Images 4–6 of Fig. 4). Y_s takes its values in $\mathbb{R}^3$ (RGB Color space) and thus, $p(y_s|u_s, x_s)$ is defined by 3-D mean vectors and 3×3 covariance matrices. Multidimensional variant of the JN-HMF is then adopted and iterative conditional estimation (ICE) [17] is used along with stochastic gradient for parameters estimation. For all experiments, $K = 2$ and $M = 2$.

In presence of shadow, the HMF may make wrong decisions at both estimation and restoration stages. For Image 1 for instance, where ω_1 corresponds to "background" and ω_1 to "writing," dark background regions may be perceived by the HMF as belonging to "writing" like the area surrounding the letter O [see Fig. 3(a).2]. The JN-HMF, on the other hand, detects the

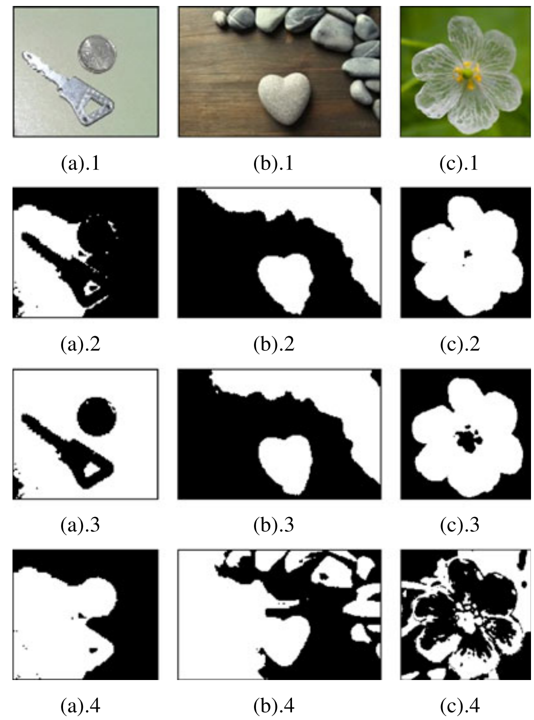

Fig. 4. Unsupervised segmentation of color images with uneven light conditions. 1. Original image; 2. HMF-based segmentation; 3. JN-HMF-based segmentation; 4. JN-HMF-based estimate of U; of (a) Image 4; (b) Image 5; (c) Image 6.

shadow, thanks to its auxiliary process [see Fig. 3(a).4], and then filters more efficiently the noise [see Fig. 3(a).3]. A similar result is obtained with Image 3. For Image 2, the HMF fails to produce satisfactory segmentation and instead, achieves what could be a good estimate of presence of shadow [see Fig. 3(b).2]. Handling separately the noise switches via the auxiliary field U, the JN-HMF yields a better segmentation [see Fig. 3(b).3].

The impact of uneven light conditions on pixel intensities is similar to the shadow one. Indeed, brightness irregularity over the image may induce the HMF to make wrong decisions. Fig. 4 illustrates the improvement made by the JN-HMF model when segmenting images with the presence of light.

IV. CONCLUSION

In this letter, we have proposed an original TMF called JN-HMF generalizing the conventional HMF by assigning to each label a set of Gaussian densities rather than only one. The choice of the appropriate Gaussian density depends on the realization of an auxiliary field U. Such process may correspond to the presence of shadow or to light conditions in the image. We have shown through experiments that the proposed JN-HMF outperforms the common hidden Markov field. An interesting future direction would be to carry out a more-elaborated comparative study to assess the performance of the JN-HMF against other approaches with application on real world data like synthetic aperture radar images where results of TMFs are very promising [18]–[21]. Another promising perspective is to consider the multisensors case where the Dempster–Shafer theory can be used to merge data as recently proposed in [22].

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