Abstract—Mobile ad-hoc networks, MANETs, are self-organized and very dynamic systems where processes have no global knowledge of the system. In this paper, we propose a model that characterizes the dynamics of MANETs in the sense that it considers that paths between nodes are dynamically built and the system can have infinitely many processes but the network may present finite stable partitions. We also propose an algorithm that implements an eventually perfect partition participant detector ♦PD which eventually detects the participant nodes of stable partitions.

Keywords—Participant detector; MANET; Partitionable networks; Models for dynamic systems;

I. INTRODUCTION

A mobile ad hoc network (MANET) is a self-organized dynamic system composed of mobile wireless nodes. Due to arbitrary failures, disconnections, arrivals, departures, or node movements, a MANET is characterized as an extremely dynamic system where links between nodes change over time. Thus, the temporal variations in the network topology imply that a MANET cannot be viewed as a static connected graph over which paths between nodes are established before the sending of a message. A path between two nodes is in fact dynamically built, i.e., a link between two intermediate nodes of a path is not necessarily established beforehand but when one node sends a message to the following one in the path. Another impact of the dynamics of MANET is that the lack of links between nodes partition them into components.

A MANET is thus a partitionable system [1], i.e., a system in which nodes that do not crash or leave the system might not be able of communicating between themselves.

Collaborative applications [2], distributed monitoring [3], resource allocation management [4] are examples of applications that support partitioning and can thus go on running in multiple partitions (components). However, such partitions must present some eventual stability (or a stability whose duration is long enough) in order to ensure that those applications can progress and terminate. When some stability conditions eventually take place for a set of processes, the latter forms a partition whose members are stable in the sense that they do neither crash nor leave the partition, and new members are not accepted. We denote such a partition a stable partition.

Motivations of the paper: The above discussion shows that there is a need for a model that takes into account the dynamics of MANETs, as well as their “stable regions”. In other words, a MANET should be modeled as a dynamic system where several stable partitions, not completely isolated, can eventually exist. Furthermore, in such a context it is essential to be able to detect the existence of such stable partitions, i.e., to provide an eventually perfect partition participant detector. Participant detectors are oracles associated with each process. The invocation of the oracle by a process gives the set of processes that belong to its partition. A participant detector can make mistakes, but if a process \( p \) belongs to a stable partition eventually and permanently, it will obtain the set of processes that are members of its partition. Similarly to failure detectors [5], the eventually perfect partition participant detector is thus characterized by both the strong partition participant completeness and eventual strong partition participant accuracy properties.

Contributions of the paper: Its contributions are twofold: (1) A model that characterizes as much as possible the behavior, dynamics, and the mentioned “stability per region” of MANETs. It also defines the conditions that the system must satisfy for supporting stable partitions; (2) an eventually perfect partition participant detector whose algorithm considers our proposed model.

II. SYSTEM MODEL

We consider a dynamic distributed system \( S \) composed of infinitely many mobile nodes. Considering one process per node, the system consists thus of an infinite countable set \( \Pi \) of processes. Contrarily to a static environment, in a dynamic anonymous system, processes do not know \( \Pi \).

To simplify the presentation of the model, we consider the existence of a discrete global clock which is not accessible to the processes. We take the range \( T \) of the clocks’ tick to be the set of natural numbers.

Processes: There is one process per node and they communicate by message-passing through an underlying wireless network. The words node and process are therefore interchangeable. Processes have unique and totally ordered identifiers, i.e., \( \forall p \in \Pi, p \) is the process identifier. A
process knows its identifier but does not necessarily know the identities of the other processes.

The topology of the network is dynamic due to node arrivals, departures, crashes, and mobility. Processes can fail by crashing. A correct process is a process that does not crash during a run; otherwise, it is faulty. A faulty node will eventually crash and does not recover.

Nodes can dynamically enter the system or leave it (voluntarily disconnect themselves from the system). A correct process that voluntarily disconnects leaves the system. A correct process that leaves the system re-enters it with a new identity and it is considered as a new process. They can also be mobile and keep continuously moving and pausing. When a node moves, its neighborhood may change and, in consequence, the set of logical links. Mobility can lead to involuntary disconnections when a process is isolated from other processes.

Processes execute by taking steps. Each process has a local clock that counts the number of steps since a fixed date. Processes are considered synchronous in the sense that we assume that there are lower and upper bounds on the rate of execution (number of steps per time unit) of any non-faulty process. Thus, to simplify our model and without loss of generality, we assume that local processing takes no time. Only message transfers take time.

Communication Links: Considering a radio propagation model, a node in a MANET communicates directly with all the other nodes that are within its transmission range. Hence, we assume that a node in a MANET never sends a point-to-point message but broadcasts a message which will be received by those nodes that are in its transmission range. If a process $q$ is within the communication range of a process $p$ we say that there is a link between $p$ and $q$. However, links between nodes are unidirectional. For instance, it might happen that a node can receive a message from another node but has insufficient remaining energy to broadcast it a message back.

We assume that our system does not modify the messages it carries, neither generate spontaneous messages nor duplicate them. Messages can be lost. Each message $m$ has a unique identifier $id_m$. The following integrity property is satisfied: $q$ receives a message $m$ from $p$ at most once only if $p$ previously sent $m$ to $q$. Messages can be delivered out of order. We define $M$ as the set of all possible messages.

Furthermore, due to node movements, lack of energy, failures, arrivals or departures, links come up and down over time. Therefore, connectivity between two nodes in MANETs is built dynamically over time as discussed in the following.

Dynamic Paths: One of the goals of our model is to define the concept of dynamic paths, i.e., the concept of end-to-end connectivity which is dynamically established through the transfer of messages along a sequence of processes.

We consider Lamport’s happened-before relation between events [6]: $a \rightarrow b$ if event $a$ causally precedes event $b$. Let $send_p(m)$ be the sending event of $m$ on process $p$ and $recv_p(m)$ be the reception event of message $m$ on $p$.

We also define $F$ a set of functions from $\Pi \times \mathcal{M}$ to $\mathcal{M}$ which associates a node processes to a message. The function outputs a message $m' = f(p, m)$ to a node $p$ and $m$. Elements of $F$ model algorithms executed by processes. Notice that the output of $f_p$ can depend upon the state of $p$.

Firstly, we define the notion of reachability: a “process $q$ is reachable from $p$ at time $t$” means that if $p$ sends a message $m$ at time $t$ then $q$ receives a message that is causally dependent upon $m$. Formally:

**Definition 1.** Reachability: $\forall (p, q, t, m, f) \in \Pi \times \Pi \times T \times M \times F, q$ is reachable from $p$ at time $t$ for the message $m$ with the algorithm $f$: if $q = p$ or if there exists $send_p(m)$ at event time $t$, then $\exists (p_1, p_2, ..., p_n) \in \Pi^n$ with $p_1 = p$ and $p_n = q$, and $\exists (m_1, m_2, ..., m_{n-1}) \in \mathcal{M}^{n-1}$ with $m_1 = m$ such that:

1. $\forall i \in [1, n-1], send_{p_i}(m_i) \rightarrow \text{recv}_{p_{i+1}}(m_i)$
2. $\forall i \in [2, n-1], m_i = f_{p_i}(m_{i-1})$

We denote $S_{p, q, t, m, f}$ the set of sequences of processes $(p_1, p_2, ..., p_n)$ that satisfy the above definition. For all $P = (p_i)_{i \in [1, n]} \in S_{p, q, t, m, f}$, we define $trec(P, t, m, f)$ the time at which $q$ receives $m_{n-1}$ and we define $mrec(P, t, m, f) = m_{n-1}$.

It is important to notice that reachability does not require the existence of an end-to-end path between $p$ and $q$ at time $t$. The path is indeed built over time.

We can now define the concept of dynamic path which will model connectivity between two processes in MANETs.

**Definition 2.** Dynamic path (denoted $p \rightarrow_q t$): $\forall (p, q, t) \in \Pi \times \Pi \times T$ there exists a dynamic path between $p$ and $q$ at time $t$: if $\forall (m, f) \in M \times F, S_{p, q, t, m, f} \neq \emptyset$.

It is worth pointing out that the reachability concept depends on the algorithm $f$ and expresses that a process can communicate with another process. On the other hand, the dynamic path concept does not depend on any algorithm. It ensures that if a process $p$ sends any message $m$, then $q$ will receive a message that causally depends on $m$. Both concepts are instantaneous, i.e., at time $t$.

We also define the concept of timely dynamic path where communication delays between processes of such a path is bounded.

**Definition 3.** Timely dynamic path (denoted $p \rightarrow_{\delta_q} t$): there exists unknown $\delta_{pq}$ such that $p \rightarrow_q t \Rightarrow \exists (m, f) \in M \times F, \exists P \in S_{p, q, t, m, f}$ such that $trec(P, t, m, f) - t < \delta_{pq}$.

Finally, we define a useful property that ensures that a node appears at most once in a timely dynamic path.
Definition 4. Simple timely dynamic path (denoted \( p^\ast q \)): \( p^m q \) and \( \exists (p_i)_{i \in [1,n]} \in S_{p,q,t,m,f} : (i \neq j \Rightarrow p_i \neq p_j) \) and \( trec((p_i)_{i \in [1,n]}, t, m, f) - t < \delta_{pq} \).

To summarize our definitions, we have \( p^m q \Rightarrow p^m q \Leftrightarrow q \) is reachable from \( p \) at time \( t \) for all messages \( m \) and all algorithms \( f \).

Eventual Group Stabilization: We denote a stable partition a group of processes which present an eventual stabilization. Basically, the stable partition of a process \( p \), denoted \( \triangledown \text{PART}_p \), is composed of the same set of correct processes that can always communicate to each other through simple timely dynamic paths. Thus, processes within \( \triangledown \text{PART}_p \) neither crash nor leave it, and new node arrivals in the partition do not take place. However, dynamic paths can evolve and processes can move inside the stable partition as long as they keep being connected by a simple timely dynamic path.

If a node \( q \) is reachable from \( p \), and vice-versa, and if \( p \) is stable, then \( q \) must be in the partition of \( p \). We define therefore the set of nodes that can be mutually reachable through a process \( p \) at a time \( t \). These nodes form cycles which include \( p \). The nodes that compose the cycles of \( p \), denoted by \( \text{Cycle}_p(t) \), are then defined as follows:

Definition 5. \( \text{Cycle}_p(t) \) is \( \{ q | \exists (m, f, n) \in \mathcal{M} \times \mathcal{F} \times N : \exists (p_i)_{i \in [1,n]} \in S_{p,q,t,m,f} : \exists k \in [1,n] : p_k = q \} \).

Definition 6. We define the stability property of a node \( p \) if there exists \( t \) such that:

1. \( \forall t' \geq t : \text{Cycle}_p(t') = \text{Cycle}_p(t) \)
2. \( \forall t' \geq t : q, r \in \text{Cycle}_p(t') \Rightarrow q e^\ast r \),
3. \( \exists N : \forall t_0 : |\text{Cycle}_p(t_0)| \leq N \)

Definition 7. A node \( p \) is \( \triangledown \text{stable} \) if \( \forall q \in \text{Cycle}_p(t), q \) has the stability property.

Now, we define the Stabilization Time of a \( \triangledown \text{stable} \) node \( p \) as the minimal time \( ST_p \) that satisfies the above definition. \( ST_p \) is unknown.

A stable partition, denoted by \( \triangledown \text{PART}_p \) of a \( \triangledown \text{stable} \) process \( p \), is defined as follows:

Definition 8. \( \triangledown \text{PART}_p \) is \( \text{Cycle}_p(ST_p) \).

Axiom 1 of Definition 6 states that the set of nodes of cycles of \( p \) does not change. This set is therefore the partition. Note that processes that belong to the partition are fixed but paths between processes can evolve during time. Axiom 2 imposes the existence of timely links between all nodes of a partition. It ensures therefore that communication time between nodes of the partition is bounded and that if a process \( p \) sends a message \( m \), then each process of the partition will receive a message which is causally dependent on \( m \). Since we make no assumption on the total number of nodes of the system, Axiom 3 fixes a bound on cycles size. Finally, Definition 7 states that a node is \( \triangledown \text{stable} \) if all nodes of its cycles have the stability property. It ensures that a process \( q \) of \( p \)'s partition is also \( \triangledown \text{stable} \) and that \( p \)'s and \( q \)'s partitions are equals.

We should remark that nodes of a stable partition are not necessarily isolated from other nodes of the network. Depending on the network connectivity, it might be the case that one or more nodes of a stable partition can send or receive messages to nodes which do not belong to their partition. On the other hand, Axiom 1 of Definition 6 ensures that if \( p \) can send messages to \( q \) through timely paths, and vice-versa, then \( q \) is in \( p \)'s partition.

Figure 1 illustrates the definition of \( \triangledown \text{stable} \) nodes. All nodes in the figure are correct and the graph evolves from state 1 to state 3 and then remains in state 3. Solid arrows correspond to timely dynamic paths, otherwise the line is dashed. Before state 3 none of the nodes are \( \triangledown \text{stable} \) as there does not exist any stable partition, i.e., there is no set of processes that satisfies Definition 6. On the other hand, as there always exist timely paths between nodes \( a, b, c \), and \( e \) after state 3, these nodes are \( \triangledown \text{stable} \) and form a partition, i.e., \( \triangledown \text{PART}_a = \triangledown \text{PART}_b = \triangledown \text{PART}_c \).

III. EVENTUALLY PERFECT PARTITION PARTICIPANT DETECTOR

Based on the system model defined in the previous section, we present in this section an algorithm for detecting the participants of a partition which implements an eventually perfect partition detector \( \text{PD} \).

Each process \( p \) has locally an eventually perfect partition participant detector, denoted \( \text{PD} \). When invoked, \( \text{PD} \) returns to \( p \) the set of processes that are mutually reachable from \( p \), i.e., those processes that it believes to belong to its partition. If \( p \) is a \( \triangledown \text{stable} \) node, eventually, \( \text{PD} \) will return the nodes that belong to the stable partition \( \triangledown \text{PART}_p \) and only these nodes.

Similarly to failure detectors, \( \text{PD} \) is characterized by both the completeness and the accuracy properties. Completeness characterizes the capability of the \( \triangledown \text{stable} \) node \( p \) of constructing an output set which contains the identification of the processes that belong to its partition while the
accuracy characterizes the capability of that process of not being included in a set of those processes which are not in its partition.

- **Strong partition participant completeness**: For each stable process $p$, if $q \in \diamond P A R T_p$, then eventually $p$ considers $q$ as a member of its stable partition permanently.
- **Eventually strong partition participant accuracy**: For each stable process $p$, if $q \not\in \diamond P A R T_p$, then eventually $p$ will no longer consider $q$ as a member of its stable partition.

**Algorithm 1** Implementation of Eventually Perfect Partition Participant Detector

```plaintext
Init:
Begin

{ Processes supposed to be in $\diamond P A R T_p$ }
inPart ← { $p$ }; output ← { $p$ };
Timeout ← $a$;
set timer to Timeout;
broadcast_nbq((ALIVE, $p$));

End

Task T1: upon reception of $(⟨ALIVE, path⟩)$
Begin
If first node in path = $p$ then
    For all $q$: $q$ appears after $p$ in path do
        inPart ← inPart $\cup$ {$q$};
    Else
        If $p$ appears at most once in path then
            broadcast_nbq((ALIVE, path $\cdot$ $p$));
End

Task T2: upon expiration of Timeout
Begin
If output $\neq$ inPart then
    Timeout ← Timeout + 1;
    output ← inPart;
    set timer to Timeout;
inPart ← {$p$};
broadcast_nbq((ALIVE, $p$));

End

Task T3: when membership() is invoked by the upper layer
Begin
    return(output);
End
```

Since processes do not know the identity of the other processes, they cannot send point-to-point messages to them. Thus, the only sending primitive provided to process $p$ is the broadcast_nbq primitive that allows $p$ to send a message to all its current neighbors (nodes within its transmission range) without necessarily knowing their identity. Due to the dynamics of the system the set of neighbors of $p$ can change during a run. A second remark is that a node $q$ that received a broadcast message from $p$ is not necessarily capable of broadcasting a message to $p$ since links are unidirectional.

Algorithm 1 implements an eventually perfect partition participant detector $\diamond PD$ for process $p$. By querying its local $\diamond PD$ (Line 30), process $p$ obtains the current knowledge of the set of processes that belong to its partition by consulting the variable output (Line 32).

The local detector executes an initialization phase and then two concurrent tasks. At the initialization phase (Lines 4–7), it initializes its timer and sends to all its neighbors an ALIVE message which includes just $p$.

Task $T_1$ handles $p$’s detector reception of an $(ALIVE, path)$ message from those processes that have $p$ as their neighbor. If path is equal to $(p, \ldots)$, $p$ knows that its $(ALIVE, p)$ message was forwarded through a cycle, i.e., all nodes that appear after $p$ in path are mutually reachable from it (Lines 13–14). Otherwise, if $p$ does not appear in path or appears just once, $p$’s detector appends $p$ to path and forwards it to all its neighbors (Lines 16–17).

Task $T_2$ is executed whenever the timeout expires. If the new set of nodes that $p$’s detector believes to belong to $p$’s partition (inPart_p) is different from the previous one, it increments the timeout value (Lines 22–23). This means that, if $p$ is a stable node, either the ST_p is not reached yet or it is reached but the timeout value is not enough for the message $(ALIVE, p)$ sent from $p$ to travel through the longest cycle from $p$. When both conditions happen, the set of processes in inPart, and thus in output, will always be the same. Finally, in Lines 24–27, $p$’s detector initializes its timer and the variable inPart, and then broadcasts to all its neighbors an ALIVE message that contains just $p$ as reachable, as in the initialization phase.

Due to lack of space, the proof that Algorithm 1 implements an eventually perfect participant partition detector $\diamond PD$ is not presented. Basically, the idea of the proof is to show that if there exists $t_1$ such that $\forall t \geq t_1$, $q \in C y c l e_p(t)$ then eventually and permanently $q \in output_p$ (strong partition completeness); otherwise, there exists a time after which $q \not\in output_p$ permanently (eventual strong partition accuracy). The full proof is available in [7].

**IV. RELATED WORK**

Modeling dynamic systems is an open issue and new models aiming at capturing different aspects of such dynamics have been defined. Like in our work, in [8], the authors state that a dynamic system must present some stability period in order to guarantee progress and termination of the computation. However, in their work, there exists just a single reliable core cluster during a period of stability which consists of the minimal number of nodes that have to be simultaneously alive during a long enough period of time in order for the whole system to be able to progress. Furthermore, the number of processes in each run is bounded and links are considered to be bidirectional.

Piergiorgianni et al. [9] also consider that a dynamic system can be characterized by perturbed periods followed by quiescent periods, i.e., periods where no more arrivals
or departures take place. The paper shows that there is no protocol that can ensure overlay network connectivity during perturbed periods since network partitions can happen.

In [10], the authors propose a model for dynamic systems where two parameters, the number of nodes (in a run or in all runs) and the diameter of the network, can be characterized (e.g., bounded/unbounded, known/unknown) depending on the dynamics of the system. The first parameter allows to model continuous arrivals and departures of nodes from the system while the second one allows to circumvent the impossibility of a node to have a global point-to-point connectivity view of the network.

Aiming at characterizing the dynamics of a system caused by the arrivals and departures of participating processes of a system, Baldoni et al. present in [11] a generic model of churn which is based on deterministic joining and departure distributions of nodes where the number of processes in the systems remains, at any time, in a given range. However, they do neither provide a means of modeling partitionable networks nor dynamic paths.

In [12], the authors have introduced the notion of evolving graphs in order to model the temporal dependency of paths in dynamic systems such as MANET or DTN (disruption tolerant networks). Concisely, an evolving graph is a time-indexed sequence of subgraphs, where the subgraph at a given time-step corresponds to the network connectivity at the time interval indicated by the time-step value. Like in our model, evolving graphs capture the notion of path over time. Nevertheless, evolving graphs are based on time-step schedulers, nodes have a global view of the connectivity graph, and path over time can not be characterized as timely. Furthermore, they do not support infinitely many nodes.

Aguilera et al. present in [13] a heartbeat failure detector, HB, for partitionable network. The output of the failure detector at each process p is an array with one entry for each process of the system. The heartbeat sequence of every process not in the same partition of p is bounded. Our partition participant detector algorithm is inspired by this work. Contrarily to our approach, in the authors’ work, the system is considered to be a fully-connected static one, the number of nodes of the system is known, nodes do not move or leave the system, and all links are fair lossy.

In a previous work [14], we have proposed an eventual partition failure detector for MANETs that uses information provided both by the above mentioned Aguilera et al.'s HB failure detector [13] and a disconnection detector. However, the number of nodes is known and the solution is neither based on periods of stability nor on dynamic paths.

V. CONCLUSION

This paper proposes a model for dynamic networks, such as MANETs, which considers that the system is anonymous with an infinite set of processes. The model characterizes the concept of dynamic paths between processes built over time as well as the concept of stable partitions, where a finite set of nodes are connected through timely dynamic paths. Based on this model, we propose an algorithm for an eventually perfect partition participant detector, \( \diamond P D \).

REFERENCES


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