Signal Design for MIMO-OFDMA Uplink

Supélec

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**INTRODUCTION**

- Consider a MIMO-OFDMA Uplink transmission
  - $K$ users transmit to a Base Station (BS)
  - All users share $N$ subcarriers following a given Carrier Assignment Scheme
  - Each user has $N_t$ transmit antennas
    The receiver (BS) has $N_r$ receive antennas

- **Reception is corrupted by**
  - an *unknown* frequency-selective propagation channel
  - an *unknown* Carrier Frequency Offset (CFO) due to
    - Doppler spreads
    - Oscillator drifts
Introduction

- **Context**: Data-aided estimation of channels and CFOs

- **Questions**
  - What is the impact of CFOs and channels estimation errors?
  - What is the performance of channels and CFOs estimation?
  - How to design training sequences that provide accurate estimates?
  - How to design high data rate systems robust to the presence of CFO?
• **Impact of Carrier Frequency Offset (CFO) impairments**
  → Example of a single-user single-carrier SISO transmission

• **Joint channels and CFO’s estimation in MIMO-OFDMA Uplink**
  → Asymptotic Cramér Rao Bound

• **Design of relevant training sequences**

• **Open problems**
Impact of Frequency Offset Impairments

- Basic example: flat fading channel
  Consider the following received complex envelope:
  \[ y(n) = e^{i\omega n} a(n) + v(n) \]
  - \( a(n) \rightarrow \) transmitted symbol sequence (variance \( \sigma^2_a \))
  - \( v(n) \rightarrow \) AWGN (variance \( \sigma^2 \))
  - \( \omega \rightarrow \) (angular) frequency offset

- How does \( \omega \) affect the Mean Square Error?
  \[ E \left[ |y(n) - a(n)|^2 \right] = \sigma^2 + 4\sigma^2_a \sin^2 \left( \frac{n\omega}{2} \right) \]
  \( \Rightarrow \) Increases with time \( n \) (phase drift \( \propto n\omega \))
IMPACT OF FREQUENCY OFFSET IMPAIRMENTS

- Example of a frequency selective channel

\[ y(n) = e^{i\omega n} \sum_{k=0}^{L-1} h_k a(n - k) + v(n) \]

\[ \triangleright h = [h_0, \ldots, h_{L-1}]^T \rightarrow \text{frequency selective channel} \]

- Training based ML estimate of \([\omega, h]^T\)

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<tr>
<th>TRAINING</th>
<th>DATA</th>
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<td>(N_T)</td>
<td>(N_D)</td>
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- Result [Ciblat-Vandendorpe,02]

\[ E\left[(\hat{\omega}_{NT} - \omega)^2\right] = O\left(\frac{1}{N_T^3}\right) \]

\[ E\left[\|\hat{h}_{NT} - h\|^2\right] = O\left(\frac{1}{N_T}\right) \quad \text{as} \ N_T \to \infty \]
**Impact of Frequency Offset Impairments**

- Assume the following (simple) receiver structure

\[
y(n) \xrightarrow{e^{-in\hat{\omega}_{NT}}} \hat{h}_{NT}(z) \xrightarrow{\hat{h}_{NT}^*(z^{-1})} z(n)
\]

1. ML parameter estimation
2. CFO compensation
3. Wiener filtering based on channel estimate

- **At time \( n \), what is the impact of estimation errors on MSE?**
**Impact of Frequency Offset Impairments**

- **Simulations.** $N_T = 50$ training symbols / $N_D = 500$ data symbols

  - $MSE(n)$ increases with time $n$
  - When $n \approx N_T$, CFO and Channel estimation errors have comparable influence
    - $\hat{h}_{NT} - h \approx \frac{1}{\sqrt{N_T}}$
    - Drift: $n(\hat{\omega}_{NT} - \omega) \approx n \frac{1}{N_T^{3/2}} \approx \frac{1}{\sqrt{N_T}}$
  - Impact of CFO predominant when $n \gg N_T$

- **Consequences.** It is crucial to
  - provide accurate estimates of $\omega$
  - take into account imperfect CFO estimation when designing systems

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OUTLINE

- Impact of Carrier Frequency Offset (CFO) impairments
  → Example of a single-user single-carrier SISO transmission
- Joint channels and CFO’s estimation in MIMO-OFDMA Uplink
  → Asymptotic Cramér Rao Bound
- Design of relevant training sequences
- Open problems
**Signal Model**

- **OFDMA Uplink:** $K$ (mobile) users share $N$ subcarriers
- **MIMO:** $N_t$ transmit antennas per user / $N_r$ received antennas

Mathematical representation:

$$
\begin{align*}
S_k^{(1)}(n) & \xrightarrow{\text{OFDM}} (t) \\
S_k^{(t)}(n) & \xrightarrow{\text{OFDM}} (t) \\
S_k^{(N_t)}(n) & \xrightarrow{\text{OFDM}} (t)
\end{align*}
$$

$$
\begin{align*}
& h_k^{(r,t)}(z) & \sigma_k \\
& \text{AWGN} & \text{AWGN}
\end{align*}
$$

$$
\begin{align*}
& \text{Receiver}
\end{align*}
$$
Signal Model

- Signal received by antenna $r = 1 \ldots N_r$

$$y^{(r)}(n) = \sum_{k=1}^{K} \left( e^{i\omega_k n} \sum_{t=1}^{N_t} \sum_{l=0}^{L-1} h_{k}^{(t,r)}(l) a_{k}^{(t)}(n-l) \right) + v^{(r)}(n).$$

- Sequence $a_{k}^{(t)}(n)$ is the IFFT of the transmitted symbols of user $k$

$$a_{k}^{(t)}(n) = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} s_{k}^{(t)}(j) e^{2i\pi \frac{n j}{N}}$$

- **Goal**: To estimate
  - all $K$ MIMO channels $\Rightarrow \{ h_{k}^{(t,r)}(l) \}_{t,r,l} \ \forall k = 1 \ldots K$
  - all $K$ frequency offsets $\Rightarrow \omega_k, \ \forall k = 1 \ldots K$
Data-Aided Estimation

- **Problem**
  Performance of estimates based on Training Sequence transmission?

- **Approach**
  Study of the Cramér-Rao Bound (CRB) for joint parameter estimation

- [Stoica, Besson, 03] → CRB for single user / single carrier / SISO transmission
- [Gault, Hachem, Ciblat, 05] → CRB for Channel and Clock offset / OFDM
One whole OFDM block is devoted to training ($\neq$ isolated pilot symbols)

\[
N = \text{size of an OFDM symbol} = \text{size of the training sequence of each user}
\]

For each user, one training sequence $s_k^{(t)}(0), \ldots, s_k^{(t)}(N-1)$ per antenna $t$

- Training sequences are transmitted in the frequency domain
- Training sequences of distinct users are independent
- Training symbols transmitted at distinct subcarriers are independent
- Possible correlation between transmit antennas

Training sequences of all users are transmitted quasi-synchronously
Cramér-Rao bound

\[ y^{(r)}(n) = \sum_{k=1}^{K} \left( e^{i\omega_k n} \sum_{t=1}^{N_t} \sum_{l=0}^{L-1} h_k^{(t,u)}(l) a_k^{(t)}(n-l) \right) + v^{(r)}(n). \]

- Stack all samples received by all antennas into vector \( y \)

\[
    y = \begin{bmatrix}
        y^{(1)}(0) \\
        \vdots \\
        y^{(1)}(N-1) \\
        \vdots \\
        y^{(N_r)}(0) \\
        \vdots \\
        y^{(N_r)}(N-1)
    \end{bmatrix} = \sum_{k=1}^{K} (I_{N_r} \otimes \Gamma(\omega_k) A_k) h_k + v
\]

\( h_k \) contains all taps \( \{ h_k^{(r,t)}(l) \}_{r,t,l} \) of user \( k \)'s channel

\( \Gamma(\omega_k) = \text{diag}(1, e^{i\omega_k}, \ldots, e^{i\omega_k(N-1)}) \)

- Matrices \( A_1, \ldots, A_K \) contain symbols \( a_k^{(t)}(n) \to \text{known (training)} \)
Cramér-Rao Bound

- Parameter vector

\[ \theta = [\omega_1, h_{R,1}, h_{I,1}, \ldots, \omega_K, h_{R,K}, h_{I,K}]^T \]

where \( h_{R,k} = \Re[h_k] \), \( h_{I,k} = \Im[h_k] \)

- Exact Cramér-Rao Bound

\[
\text{CRB}_N = \left( \frac{2}{\sigma^2} \Re \left[ \frac{\partial \left( \sum_{k=1}^{K} (I_{N_r} \otimes \Gamma(\omega_k)A_k) h_k \right)^H}{\partial \theta} \frac{\partial \left( \sum_{k=1}^{K} (I_{N_r} \otimes \Gamma(\omega_k)A_k) h_k \right)}{\partial \theta^T} \right] \right)^{-1}
\]

\( \Rightarrow \) complicated, non-informative
Asymptotic Cramér-Rao Bound

Number $N$ of subcarriers tends to infinity

- As $N \to \infty$, a compact expression of $\text{CRB}_N$ can be obtained.

- **Validity of the results**
  When $N$ is significantly larger than the number $K(1 + 2LN_r N_t)$ of parameters
  \[ \Rightarrow N \gg K \]

- **Remark**
  The total bandwidth remains constant
  \[ \Rightarrow \text{When } N \to \infty, \text{ subcarrier spacing } \frac{1}{N} \text{ tends to zero} \]
**Asymptotic Cramér-Rao Bound**

- **Case of a single antenna per user** = single training sequence $s_{N,k}(j)$
  
  For each user $k$, define the following measure for any set $A$ of $[0, 1]$
  
  $$
  \mu_{N,k}(A) = \frac{1}{N} \sum_{j=0}^{N-1} E \left[ |s_{N,k}(j)|^2 \right] I_A \left( \frac{j}{N} \right)
  $$

  **Assumption:** When $N \to \infty$, $\mu_{N,k}$ converges weakly to a limit measure $\mu_k$.

  - As subcarrier spacing $\frac{1}{N} \to 0$, $\mu_{N,k}(A)$ tends to constant $\mu_k(A)$
  
  - Density of $\mu_k = \text{limit variance profile}$
Asymptotic Cramér-Rao Bound

- **Case of** $N_t$ **antennas per user** = $N_t$ **training sequences** $s_{N,k}^{(t)}(j)$
  For each user $k$, for each couple of antennas $(t, t')$, define

  \[
  \mu_{N,k}^{(t,t')} (A) = \frac{1}{N} \sum_{j=0}^{N-1} E \left[ s_{N,k}^{(t)}(j)s_{N,k}^{(t')}^{(j)*} \right] I_A \left( \frac{j}{N} \right)
  \]

- **Assumption:** When $N \to \infty$, $\mu_{N,k}^{(t,t')}$ converges weakly to a limit measure $\mu_{k}^{(t,t')}$.

- **Interpretation**

  \[
  \mu_k(A) = \begin{bmatrix}
  \mu_k^{(1,1)}(A) & \cdots & \mu_k^{(1,N_t)}(A) \\
  \vdots & & \vdots \\
  \mu_k^{(N_t,1)}(A) & \cdots & \mu_k^{(N_t,N_t)}(A)
  \end{bmatrix},
  \]

  can be interpreted as the covariance profile between antennas of user $k$
Why to introduce these covariance profiles $\mu_1, \ldots, \mu_K$?

- For each $k$, $\mu_k$ encompasses
  - the power allocation policy
  - the Carrier Assignment Scheme (CAS)
  - the correlation between antennas (beamforming)
- The asymptotic performance only depends on $\mu_1, \ldots, \mu_K$
  $\Rightarrow$ no further assumptions on the particular training strategy is required
**Asymptotic Cramér-Rao Bound**

- **Result**

  Define $\mathbf{W}_N = \text{diag}(\mathbf{w}_N^T, \ldots, \mathbf{w}_N^T)$ where $\mathbf{w}_N = [N^{3/2}, N^{1/2}, \ldots, N^{1/2}]$.

  The CRB for parameter $\mathbf{\theta} = [\omega_1, \mathbf{h}_{R,1}, \mathbf{h}_{I,1}, \ldots, \omega_K, \mathbf{h}_{R,K}, \mathbf{h}_{I,K}]^T$ is such that

  $$
  \mathbf{W}_N \mathbf{\text{CRB}}_N \mathbf{W}_N \xrightarrow{P} \text{diag}(\mathbf{C}_1, \ldots, \mathbf{C}_K)
  $$

  where each block $\mathbf{C}_k$ is equal to

  $$
  \frac{\sigma^2}{2} \begin{bmatrix}
  \frac{12}{\gamma_k} & \frac{6}{\gamma_k} \mathbf{h}_{I,k}^T & -\frac{6}{\gamma_k} \mathbf{h}_{R,k}^T \\
  \frac{6}{\gamma_k} \mathbf{h}_{I,k} & \Re \left[ \mathbf{I}_{N_r} \otimes \mathbf{R}(\mu_k)^{-1} \right] + \frac{3}{\gamma_k} \mathbf{h}_{I,k} \mathbf{h}_{I,k}^T & -\Im \left[ \mathbf{I}_{N_r} \otimes \mathbf{R}(\mu_k)^{-1} \right] - \frac{3}{\gamma_k} \mathbf{h}_{I,k} \mathbf{h}_{R,k}^T \\
  -\frac{6}{\gamma_k} \mathbf{h}_{R,k} & -\Im \left[ \mathbf{I}_{N_r} \otimes \mathbf{R}(\mu_k)^{-1} \right] - \frac{3}{\gamma_k} \mathbf{h}_{R,k} \mathbf{h}_{I,k}^T & \Re \left[ \mathbf{I}_{N_r} \otimes \mathbf{R}(\mu_k)^{-1} \right] + \frac{3}{\gamma_k} \mathbf{h}_{R,k} \mathbf{h}_{R,k}^T
  \end{bmatrix}
  $$

  - $\mathbf{R}(\mu_k)$ is a simple function of $\mu_k$

  - $\gamma_k = \mathbf{h}_k^H (\mathbf{I}_{N_r} \otimes \mathbf{R}(\mu_k)) \mathbf{h}_k$ coincides with received power of the signal transmitted by user $k$. 

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**Consequences**

- Asymptotic CRB = block-diagonal matrix

\[ \Rightarrow \text{for distinct users } k \neq k', \text{ estimation errors on } \omega_k, \mathbf{h}_k / \omega_{k'}, \mathbf{h}_{k'} \text{ are uncorrelated} \]

- For any unbiased estimate of \( \theta \) and for large values of \( N \),

\[
E \left[ (\hat{\omega}_{N,k} - \omega_k)^2 \right] \geq \frac{1}{N^3} \frac{6\sigma^2}{\gamma_k} \\
E \left[ \| \hat{\mathbf{h}}_{N,k} - \mathbf{h}_k \|^2 \right] \geq \frac{1}{N} \left( \sigma^2 \text{tr} \left( \mathbf{R}(\mu_k)^{-1} \right) + O_L(1) \right)
\]

Respectively denote by \( \frac{1}{N^3} CRB_{\omega,k} \) and \( \frac{1}{N} CRB_{h,k} \) the above bounds

- Asymptotic performance does not depend on the number of users

- For a given user \( k \), asymptotic performance does not depend on the parameters of other users \( l \)
CONSEQUENCES

- Assume $N_t = 1 \rightarrow$ Asymptotic performance only depends on $\mu_k$
- Consider the two following training strategies

\begin{itemize}
  \item T1: “Full” Carrier Assignment
  \item T2: Interleaved OFDMA
\end{itemize}

- Both training strategies have the same asymptotic variance profile $\mu_k$

$\Rightarrow$ Orthogonal and non-orthogonal CAS lead to identical asymptotic performance.
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- Design of relevant training sequences

- Open problems
Training Sequence Design

- **Question**: What training strategies are likely to provide accurate estimates?
- For each user, $N_t$ training sequences have to be designed (one per antenna)
- **Idea**
  - Minimize $CRB_{\omega,k}$ to obtain an accurate estimate of the CFO
  - Minimize $CRB_{h,k}$ to obtain an accurate estimate of the channel
- **Remark**
  For a given user $k$, $CRB_{\omega,k}$ and $CRB_{h,k}$ only depend on $\mu_k, h_k$
  $\Rightarrow$ Training strategy for user $k$ does not involve other users $l \neq k$
Optimal Training for CFO Estimation

(One can drop index $k$)

- **Problem**
  Find covariance profile

$$\bm{\mu}(A) = \begin{bmatrix} 
\mu^{(1,1)}(A) & \cdots & \mu^{(1,N_t)}(A) \\
\vdots & \ddots & \vdots \\
\mu^{(N_t,1)}(A) & \cdots & \mu^{(N_t,N_t)}(A) 
\end{bmatrix}$$

which minimizes

$$CRB_\omega = \frac{6\sigma^2}{\gamma}$$

under transmit power constraint

$$\text{tr} (\bm{\mu}([0, 1])) \leq P$$
Optimal Training for CFO Estimation

- Denote by

\[
\mathbf{h}^{(r)}(f) = \sum_{l=0}^{L-1} \begin{bmatrix}
    h^{(r,1)}(l) \\
    \vdots \\
    h^{(r,N_t)}(l)
\end{bmatrix} e^{2i\pi lf}
\]

the \(N_t \times 1\) frequency response of the channel “seen” by antenna \(r\)

- Equivalent problem: \(CRB_\omega\) can be written

\[
CRB_\omega = \frac{6\sigma^2}{\sum_{r=1}^{N_r} \int_0^1 \mathbf{h}^{(r)}(f)^H \mu(df) \mathbf{h}^{(r)}(f)}
\]
Optimal Training for CFO Estimation

- Resulting training strategy

For each \( f \), denote by \( \lambda(f) \) the largest eigenvalue of \( \sum_{r=1}^{N_r} h^{(r)}(f) h^{(r)}(f)^H \).

Define

1. \( f_{opt} = \arg\max_f \lambda(f) \)
2. \( v_{opt} = \text{eigenvector associated with } \lambda(f_{opt}) \)

\[ \Rightarrow \text{CRB}_\omega \text{ is minimum for } \mu(A) = \mathfrak{P} v_{opt} v_{opt}^H \delta_{f_{opt}}(A) \]

- In practice

1. Send maximum power at the “best” frequency \( \rightarrow f_{opt} \)
2. Introduce appropriate correlation between antennas \( \rightarrow v_{opt} \)

- This does not allow to properly estimate the channel!
Optimal Training for Channel Estimation

- Assumptions
  - $L$ is large (but $N \gg L$)
  - Measure $\mu$ has a density, i.e. $\mu(df) = P(f)df$

- Problem
  
  Minimize $CRB_h \approx \sigma^2 \text{tr} \left( R(\mu)^{-1} \right)$ under power constraint $P$

- Resulting training strategy: $P(f) = \frac{P}{N_t} I_{N_t}$
  1. Uniform power profile
  2. No correlation between antennas

- Remark
  No training strategy is jointly optimal for CFO and channel estimation
OUTLINE

• **Impact of Carrier Frequency Offset (CFO) impairments**
  → Example of a single-user single-carrier SISO transmission

• **Joint channels and CFO’s estimation in MIMO-OFDMA Uplink**
  → Asymptotic Cramér Rao Bound

• **Design of relevant training sequences**

  ▶ **Open problems**
Open Problem #1

- Need of a Training Design criterion
  
  No training sequence is jointly optimal for channel and CFO estimation
  
  $\Rightarrow$ What tradeoff between CFO and channel estimation accuracy?
  
  $\Rightarrow$ Need a relevant criterion

- A possible idea

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<td>n=1</td>
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  MSE increases with time $n$

  What training sequence allows to transmit the largest number of data blocks, while maintaining a predefined QoS (typically $MSE(n) \geq MSE_{min}$)?
Open Problem #2

- Knowledge of Channel’s statistics only
  - Optimal training design rules depends on the particular channel
  - In certain applications, CSI is limited to some channel’s statistics:
    - Correlation in time of channel taps $h^{(r,t)}(0), \ldots, h^{(r,t)}(L - 1)$
    - Correlation in space, between antennas

- Problem
  - Minimize $E_h[CRB_\omega]$
  - What performance gain using a “colored” training sequence?
Open Problem #3

- **Signal Design for time-variant channels**
  - In practice, Doppler spread $\Rightarrow$ time-variant channel
  - Knowledge of CFO $\Rightarrow$ knowledge of coherence time
    
    How to exploit this knowledge?

- **Related problems**
  1. Using our CFO estimate,
     - How frequently should we send training sequences so as to keep a QoS?
     - Where to place training blocks?
  2. Users have different Doppler spreads $\Rightarrow$ Different coherence times
     $\rightarrow$ Investigate asynchronous training sequence transmission
Simulation Results

SNR, dB

CRB (N=64)

$N_t = 1, \ N_r = 1, \ White$

$N_t = 1, \ N_r = 1, \ Optimal$

$N_t = 2, \ N_r = 1, \ White$

$N_t = 2, \ N_r = 1, \ Optimal$

$N_t = 4, \ N_r = 1, \ White$

$N_t = 4, \ N_r = 1, \ Optimal$

$N_t = 4, \ N_r = 4, \ White$

$N_t = 4, \ N_r = 4, \ Optimal$

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