Capacity Requirements in Networks of Quantum Repeaters and Terminals

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Outline of the work

- Topic: Path congestion avoidance in networks of quantum repeaters and terminals
- Assumption: Complete paths between terminals
- What is the required quantum memory size in repeaters?
- Contributions:
  - Lower and upper bounds for the required qubit memory size of repeaters for general graphs and two-dimensional grid network topologies
  - Congestion avoidance algorithm: Layer-peeling path establishment
Repeater $r$ capacity

$C_P(r)$ is the number of supported paths
Simple error model: single qubit errors in Bell-EPR pairs
Achieve fidelity with purification
Adjacent nodes use direct communications to establish entanglement
Remote nodes use entanglement swapping and teleportation
Quantum memory size of a repeater is equal to the sum of the lengths of the paths going through it (Lemma 7)
For each simulation, we compute the following metrics

- **Congestion**: \# of paths passing through most visited repeater

- **Entanglement rate**: Following existing work (cf. [24,25,26])

\[
\mathcal{T}(n) = \begin{cases} 
1/R(n), & \text{if } \mathcal{X}_{ch} \geq \tau(n) - (\mathcal{X}_s - \tau(1)) \\
0, & \text{else}
\end{cases}
\]

(precise calculation is summarized in the paper)
General Graphs

- Minimum required quantum memory (Corollary 9)

\[ M_P(r) \geq 2 \left\lceil \frac{1}{|R|} \left(\frac{|T|}{2}\right) \right\rceil \text{ qubits} \]

- Maximum required quantum memory (Lemma 10)

\[ M_P(r) \leq \delta \left(\frac{|T|}{2}\right) \text{ qubits} \]

where \( \delta \) is the diameter of the graph.
In general, the quantum memory required by a repeater $r$ (Corollary 16)

$$M(r) \in \Omega(k^2) \text{ qubits.}$$
Simulation Results

- Assumption 1: Path establishment for all terminals
  - End-to-end paths from every terminal to any other terminal:

- Assumption 2: Random arrangement of repeaters using Bernoulli bond percolation
  - Probability $p$ of ensuring repeater connectivity greater than 0.5

- NetworkX library\(^1\) to conduct Monte Carlo simulations\(^2\)
- A (step-by-step) construction example follows

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\(^1\) Python Library available online at: https://networkx.github.io
\(^2\) Code available online at: http://j.mp/QCECodeGitHub
## Initial Parameters

- \( k = 20 \) #k quadratic (2D) lattice
- \( p = 1 \) #Bernoulli probability for bond percolation
- \( q = 1 \) #Bernoulli probability for terminal arrival

- DrawGrid=True
- ShowLabels=False
- AdditionalRing=True
- BondPercolation=False
- ComputePaths=False
- PathSearchAlgorithm=1 #1=shortestPaths 2=peelingPaths
- CSVFormat=False

Output:
The graph contains 324 repeaters and 72 terminals [(k^2 nodes 0, 19, 380, and 399 removed, to avoid terminal adjacency)]
## Initial Parameters

```
#k quadratic (2D) lattice
k = 20

#bernoulli probability for bond percolation
p = 0.55

#bernoulli probability for terminal arrival
q = 1
```

DrawGrid=True
ShowLabels=False
AdditionalRing=True
BondPercolation=True
ComputePaths=False
PathSearchAlgorithm=1 #1=shortestPaths 2=peelingPaths
CSVFormat=False

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Output:
The graph contains 254 repeaters and 105 terminals.

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Run 1
## Initial Parameters

\[ k = 20 \] #\( k \) quadratic (2D) lattice
\[ p = 0.55 \] #bernoulli probability for bond percolation
\[ q = 1 \] #bernoulli probability for terminal arrival

DrawGrid=True
ShowLabels=False
AdditionalRing=True
BondPercolation=True
ComputePaths=False
PathSearchAlgorithm=1 #1=shortestPaths 2=peelingPaths
CSVFormat=False

Run 2

Output:
The graph contains 266 repeaters and 108 terminals.
### Initial Parameters

- $k = 10$  # Quadratic (2D) lattice
- $p = 0.65$  # Bernoulli probability for bond percolation
- $q = 1$  # Bernoulli probability for terminal arrival

- DrawGrid=True
- ShowLabels=True
- AdditionalRing=True
- BondPercolation=True
- ComputePaths=True
- PathSearchAlgorithm=1  # 1=shortestPaths 2=peelingPaths
- CSVFormat=False

### Output:

The graph contains 56 repeaters {11, 12, 13, 14, 15, 16, 17, 18, 21, 23, 24, 25, 26, 27, 28, 31, 32, 33, 34, 36, 37, 38, 41, 42, 43, 45, 46, 47, 48, 51, 52, 53, 54, 55, 57, 58, 61, 62, 63, 67, 68, 71, 72, 73, 74, 75, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88} and 37 terminals {1, 2, 3, 4, 5, 6, 7, 8, 10, 19, 20, 29, 30, 39, 40, 49, 50, 59, 60, 69, 70, 79, 80, 89, 91, 92, 93, 94, 95, 96, 97, 98, 22, 35, 44, 64, 76}.

### Paths:

- 1 -> 2 : [1, 11, 12, 2]
- 1 -> 3 : [1, 11, 21, 31, 32, 33, 34, 24, 14, 13, 3]

...  
22 -> 35 : [22, 21, 31, 41, 42, 43, 53, 54, 55, 45, 35]  
22 -> 44 : [22, 21, 31, 41, 42, 43, 44]  
22 -> 64 : [22, 21, 31, 41, 51, 61, 62, 63, 64]  
22 -> 76 : [22, 21, 31, 41, 51, 61, 62, 63, 73, 74, 75, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88]  
35 -> 44 : [35, 45, 55, 54, 53, 43, 44]  
35 -> 64 : [35, 45, 55, 54, 53, 52, 51, 61, 62, 63, 64]  
35 -> 76 : [35, 45, 55, 54, 53, 52, 51, 61, 62, 63, 73, 74, 75, 77, 78, 81, 82, 83, 84, 85, 86, 87]  
44 -> 64 : [44, 43, 42, 41, 51, 61, 62, 63, 64]  
44 -> 76 : [44, 43, 42, 41, 51, 61, 71, 72, 73, 74, 75, 77, 78, 81, 82, 83, 84, 85, 86, 87]  
64 -> 76 : [64, 63, 73, 74, 75, 77, 78, 81, 82, 83, 84, 85, 86, 87]  

**Congestion** = 288 (Repeater 31 appears in 288 paths, repeater 41 appears in 245 paths, repeater 51 appears in 223 paths, etc.)  
**Entanglement rate** = 200
Congestion Results

Fig. 10. Congestion results using (a,c) shortest path and (b,d) peeling path strategies. Values of \( p \) and \( q \) are 0.95 in (a,b) and 0.65 in (c,d).

(a,c) shortest path and (b,d) peeling path strategies. Values of \( p \) and \( q \) are 0.95 in (a,b) and 0.65 in (c,d).
Entanglement Rate Results

Fig. 11. Entanglement rate results using (a,c) shortest path and (b,d) peeling path strategies. Values of $p$ and $q$ are 0.95 in (a,b) and 0.65 in (c,d). Values of $p$ and $q$ are 0.95 in (a,b) and 0.65 in (c,d).
Topic: Path congestion avoidance in networks of quantum repeaters and terminals

Assumption: Complete paths between terminals

Evaluation
  - shortest-path establishment vs. layer-peeling path establishment

Main results:
  - Both strategies provide an equivalent entanglement rate
  - Layer-peeling establishment considerably reduces congestion
    → Repeaters in the inner layers get less congested and would require a lower number of qubits, while providing a similar entanglement rate
References

